

Homework 3 - Strains and Deformation

Handed out: Thurs., 11-10-2007

Due to: Monday, 22-10-2007

1. Take a unit cube of material. Rotate it 90 degrees in the clockwise direction around the z-axis. Calculate the strains (both using large and infinitesimal theory). Discuss your results - their accuracy and the reasons for your conclusions.

2. Let \mathcal{B} be a body occupying the unit cube in the reference configuration (e.g.: $X, Y, Z \in [0, 1]$). The mapping between the current and the reference configuration given by $x = X + kY; y = Y; z = Z$. Find: (a) Sketch current and deformed configurations. (b) Show that motion is isochoric. (c) Find stretches in the directions \underline{e}_1 and \underline{e}_2 . (d) Find the shear strain between these both directions by using the large deformation theory and compare it to the one obtained by using the linearized strain tensor. Discuss the results taking into consideration the value of k . (e) Find principal stretches and principal directions of stretch ($k = 4$).

3. A 3-D deformation of a uniform bar stretched under its own weight gives the following linearized strain tensor:

$$[\epsilon] = \begin{bmatrix} AZ & 0 & 0 \\ 0 & AZ & 0 \\ 0 & 0 & BZ \end{bmatrix}$$

where A and B are constants and Z stands for the coordinate in the vertical direction. Determine the corresponding displacement field and identify (if they exist) the rigid body motion terms.