# HW3 - SOLUTIONS

1- $[F] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & +1 \end{bmatrix}$ ( IN THE PRESENT CASE $U = I$ OR $F = R$ )

THEN $F^T F = I$ AND $E = 0$

BUT $\underline{u} = (y - x)\,\underline{e}_1 + (-x - y)\,\underline{e}_2$

$$[\varepsilon] = \frac{1}{2}\begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \neq 0 \quad (?!)$$

PLEASE NOTE THAT FOR A ROTATION OF $\theta$ AROUND z, F WOULD TAKE THE FORM OF

$$F = R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

WHICH IMPLIES $1 + \dfrac{du_x}{dx} = \cos\theta$ ; FOR SMALL $\theta$, $\cos\theta \sim 1$

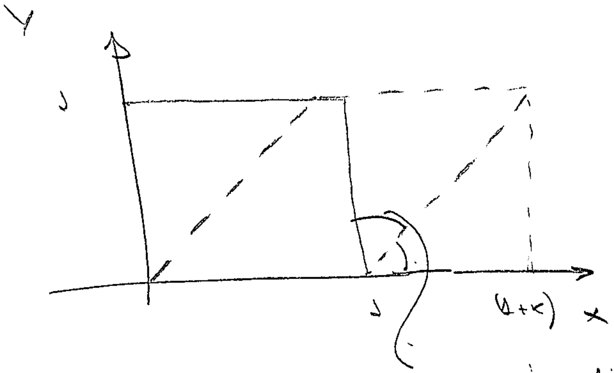
$\rightarrow \dfrac{du_x}{dx} \sim 0$ (SMALL DEF.) , BUT IF $\theta = 90°$ THAN

$\dfrac{du_x}{dx} \sim -1 \rightarrow$ LARGE DEFORMATIONS

HW3.2

2 -

(a) DEFORMATION ONLY TAKES PLACE ON THE PLANE XY ($z = 2$)




$\beta \longrightarrow \tan\beta = k$

(b) $F = \begin{bmatrix} 1 & k & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \boxed{\det F = 1}$

HOMOGENEOUS DEFORMATION !

(c) $F^T F = \begin{bmatrix} 1 & k & 0 \\ k & (k^2+1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

STRETCH $\underline{e}_1$ DIRECTION $\quad E\,\underline{e}_1 \cdot \underline{e}_1 = 0$

STRETCH $\underline{e}_2$ DIRECTION $\quad E\,\underline{e}_2 \cdot \underline{e}_2 = \frac{1}{2} k^2$

HW3.3

(d) $E_{12} = \frac{1}{2} k$

$$\varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) = \frac{1}{2} k$$

So $E_{12} = \varepsilon_{12} \quad \forall k$

(e) $$E \phi_i = \lambda_i \phi_i \Longrightarrow U^2 \phi_i = \underbrace{(2\lambda_i - 1)}_{\Lambda_i} \phi_i$$

CHARACTERISTIC EQUATION : $(1-\Lambda_i)^2\left[(k^2+1)-\Lambda_i\right] - k^2(1-\Lambda_i) = 0$

THUS $\Lambda_i$ $\begin{cases} 1 \\ \dfrac{k^2+2 \pm \sqrt{(k^2+2)^2 - 4(k^2+2)}}{2} \end{cases}$ $\begin{cases} \dfrac{9+\sqrt{252}}{2} \\ \dfrac{9-\sqrt{252}}{2} \end{cases}$

$(k=4)$

THEN $\lambda_i = \begin{cases} 1 \\ \dfrac{4+\sqrt{252}}{4} \\ \dfrac{4-\sqrt{252}}{4} \end{cases}$

PRINCIPAL DIRECTIONS

$$\lambda_{II} = 1 \longrightarrow \phi_{II} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda_{I} = 4 + \frac{\sqrt{252}}{4} \longrightarrow \phi_{I} = \begin{bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \\ 0 \end{bmatrix}$$

$$\lambda_{III} = 4 - \frac{\sqrt{252}}{4} \longrightarrow \phi_{II} = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}$$

HW 3.5

3) NON-HOMOGENEOUS DEFORMATION FIELD

$$\frac{\partial u_x}{\partial x} = A z \; ; \qquad \frac{\partial u_y}{\partial y} = A z \qquad \text{AND} \qquad \frac{\partial u_z}{\partial z} = B z$$

So :

$$u_x = A z x + C_x(y, z)$$

$$u_y = A z y + C_y(x, z)$$

$$u_z = \frac{B z^2}{2} + C_z(x, y)$$

Moreover :

$$\frac{\partial C_x}{\partial y} + \frac{\partial C_y}{\partial x} = 0$$

$$\frac{\partial C_z}{\partial x} + \frac{\partial C_x}{\partial z} = 0$$

$$\frac{\partial C_y}{\partial z} + \frac{\partial C_z}{\partial y} = 0$$

HW 3.6

AS

$$\frac{\partial C_x}{\partial Y} = -\frac{\partial C_y}{\partial X}$$

$C_x$ MUST BE LINEAR IN $Y$ AND $C_y$ MUST BE LINEAR IN $X$, THEREFORE

$$C_x(Y,Z) = aY + bZ + \ell YZ$$

$$C_y(X,Z) = cX + dZ + mXZ$$

ANALOUSLY

$$C_z(Y,X) = eY + fX + nYX$$

MOREOVER,

$$a + \ell Z = -c - mZ$$

$$\Downarrow$$

$$\boxed{c = -a}$$

$$\boxed{m = -\ell}$$

HW3.7

- $f + mY = -b - \ell Y$

$$\boxed{f = -b} \quad \text{AND} \quad \boxed{m = -\ell}$$

- $d + mX = -e - nX$

$$\boxed{e = -d} \quad \text{AND} \quad \boxed{m = -n}$$

BUT $\quad m = n \quad$ AND $\quad m = -n$

$$\Downarrow$$

$$m = n = \ell = 0$$

$$C_x(Y, Z) = aY + bZ$$

$$C_y(X, Z) = -aX + dZ$$

$$C_z(Y, X) = -dY - bX$$

HW3.8

THEN

$$\underline{u} = \underline{u}_F + \underline{u}_R(a, b, d) \qquad \forall\, a, b, d \in \mathbb{R}$$

AND $\nabla \underline{u}_R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow$ NON DEFORMATION

$\Downarrow$

RIGID BODY MOTION