Homework 6 - Plane Problems

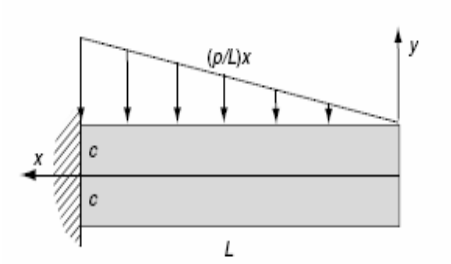
Handed out: Tue., 27-10-2007 **Due to: Thurs., 17-12-2007**

Question 1 - Establish a condition involving parameters A, B and C such that the function bellow satisfy the bi-harmonic equation and then can be used as an Airy stress function

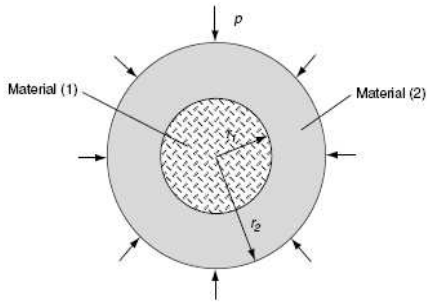
$$\phi = Ax^4 + BX^2Y^2 + CY^4$$

Question 2 - The following stress function is proposed for solving the problem of a planar beam (XY plane) clamped at the left extremity (which means that no displacements are allowed, or when stress formulation are used only resultants, moment and normal force are to be considered) and loaded by a uniformly varying force as depicted in the figure bellow. Verify if ϕ satisfies all the conditions and compute the parameters.

$$\phi = C_1XY + C_2\frac{X^3}{6} + C_3\frac{X^3Y}{6} + C_4\frac{XY^3}{6} + C_5\frac{X^3Y^3}{9} + C_6\frac{XY^5}{20}$$



Question 3 - A long composite cylinder is submitted to an external pressure as depicted bellow. Assuming that bonding between the two materials is perfect, normal stresses and displacements are continuous across the interface $r = r_1$. Determine the stress and displacement fields in the cylinder (hint: remember the stress function we have employed in a previous example).



Question 4 - As we have seen before, the typical Airy function for axisymmetric cases is given by $\phi = A + Bln(r) + Cr^2 + Dr^2ln(r)$. Compute the stress and displacement fields (if it is the case, explicitly point out the possible rigid body motions)

Question 5 - In polar coordinates, the Airy stress function $\phi = B\theta$ is used to find the solution of disk of radius a subject to a moment M acting parallel to the disk axis (therefore perpendicular to the plane of the disk). Derive the stress field and compute B in terms of M.