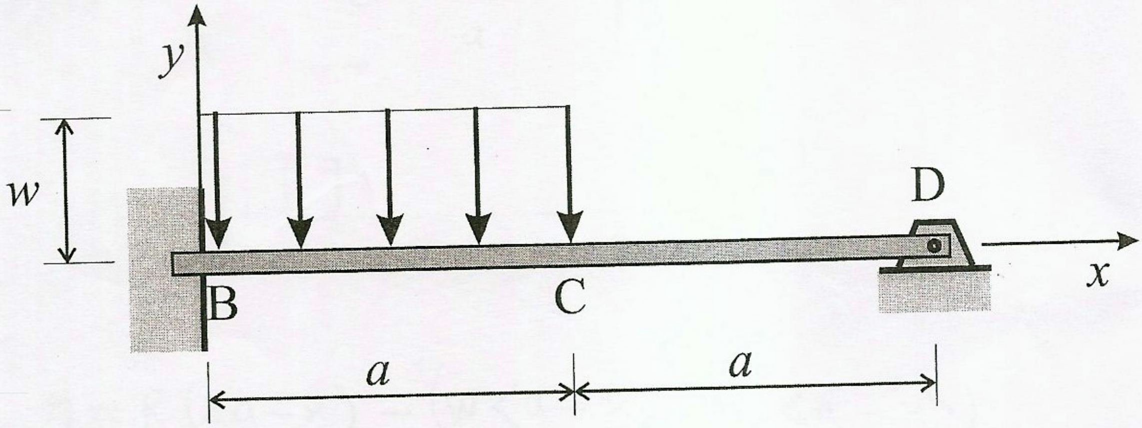
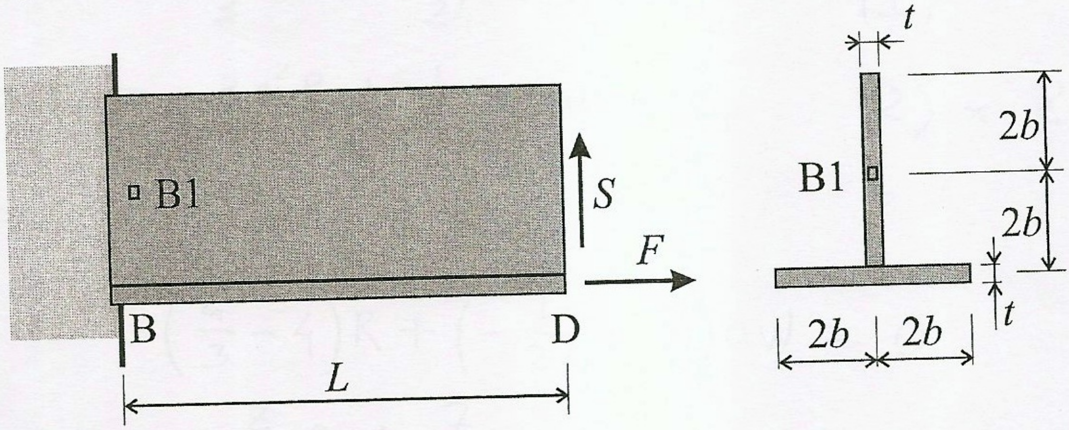
P1 - Mecânica dos Sólidos II - 2008.2 22 de Setembro, 2008

(I)



1. Calcular a reação R do apoio em D, em função de a e w.
2. Para $a = 1\,\text{m}$ e $w = 64\,\text{kN/m}$:
   (a) Desenhar os gráficos do cortante $V(x)$ e do momento fletor $M(x)$.
   (b) Calcular os momentos máximo e mínimo.

(II)



Dados: b, $t << b$, L, F e S.
Calcular:

i) o momento fletor M_B na seção B.

ii) as componentes de tensão no ponto indicado B1 da seção B, e representar estes resultados em um desenho.

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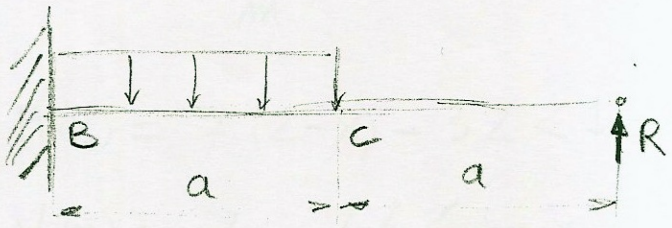
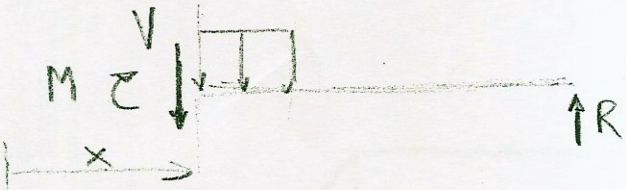
I)





$$M = R(2a - x) - \left(w\langle a - x\rangle\right)\left(\tfrac{1}{2}\langle a - x\rangle\right)$$

$$EIu'' = M = R(2a - x) - \tfrac{1}{2} w \langle a - x\rangle^2$$

$$EIu' = -\tfrac{1}{2} R(2a - x)^2 + \tfrac{1}{6} w \langle a - x\rangle^3 + c_1$$

$$EIu = \tfrac{1}{6} R(2a - x)^3 - \tfrac{1}{24} w \langle a - x\rangle^4 + c_1 x + c_2$$

$$u(0) = 0 = \tfrac{4}{3} a^3 R - \tfrac{1}{24} a^4 w + c_2 \qquad (1)$$

$$u'(0) = 0 = -2a^2 R + \tfrac{1}{6} a^3 w + c_1 \qquad (2) \times 2a$$

$$u(2a) = 0 = 2a c_1 + c_2$$

$$\left(\tfrac{4}{3} - 4\right)R + \left(-\tfrac{1}{24} + \tfrac{1}{3}\right)aw = 0$$

$$-\tfrac{8}{3} R + \tfrac{7}{24} aw = 0$$

$$R = \tfrac{7}{64} aw = 0.11\, aw$$

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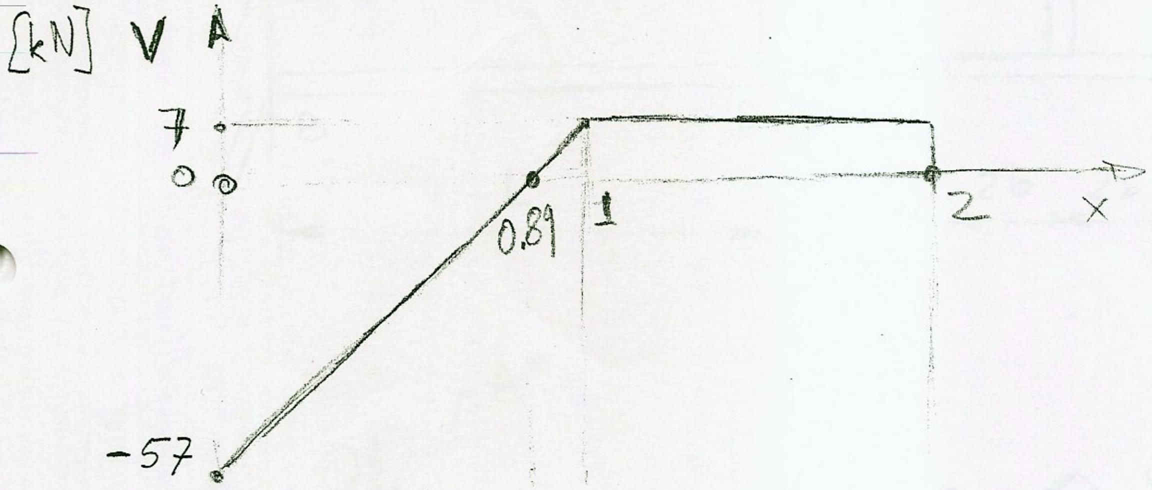
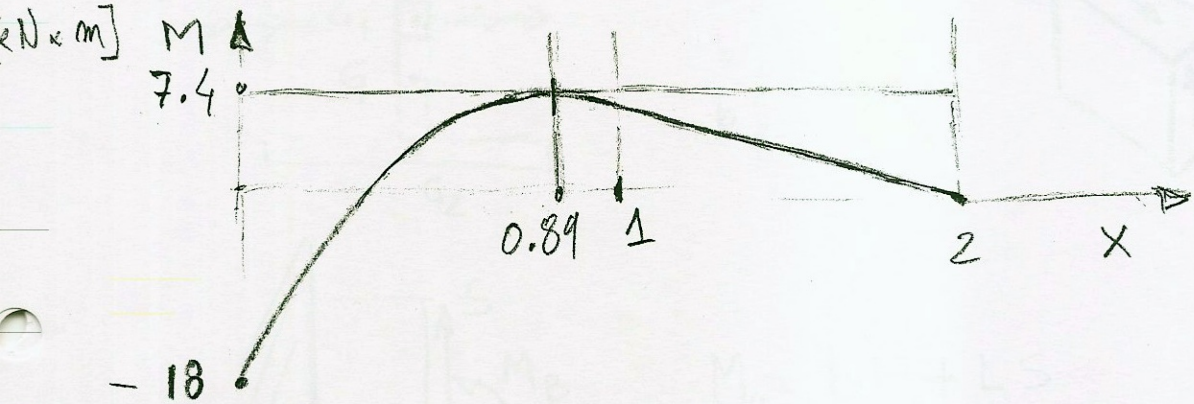
$$w = 64 \ \frac{kN}{m} \qquad a = 1 \ m \quad \Rightarrow \quad R = 7 \ kN$$

$$M(x) = 7(2-x) - 32\langle 1-x \rangle^2 \qquad [kN \times m]$$

$$V(x) = 7 - 64\langle 1-x \rangle \qquad [kN]$$





$$V = 0 \quad \Rightarrow \quad x = \frac{57}{64} = 0.891 \ m$$

$$M\left(\frac{57}{64}\right) = 7\left(2 - \frac{57}{64}\right) - 32\left(1 - \frac{57}{64}\right)^2$$

$$= \frac{7 \times 71}{64} - \frac{7^2}{2 \times 64} = \frac{7}{2 \times 64}(142 - 7)$$

$$= \frac{7 \times 135}{2 \times 64} = \frac{945}{128} = \mathbf{7.38} \qquad [kN \times m]$$

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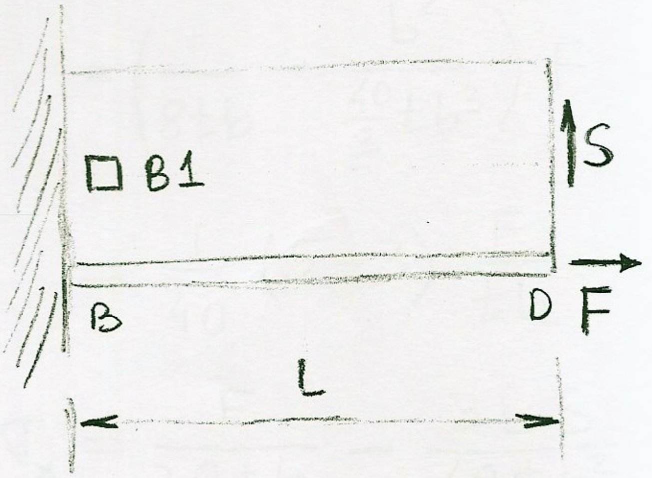
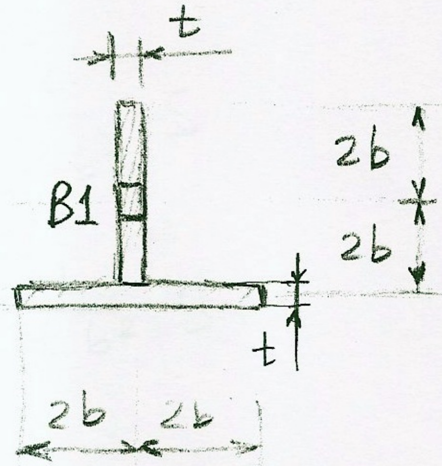
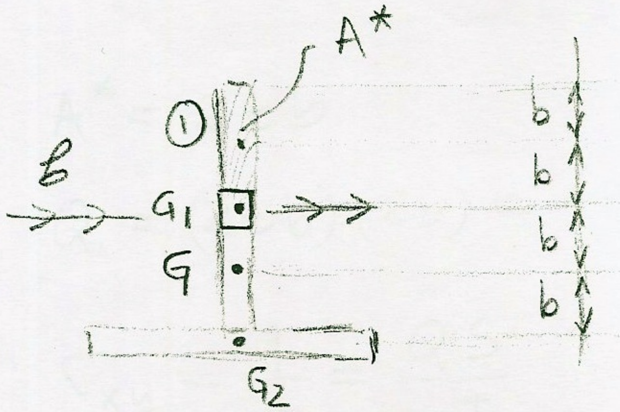
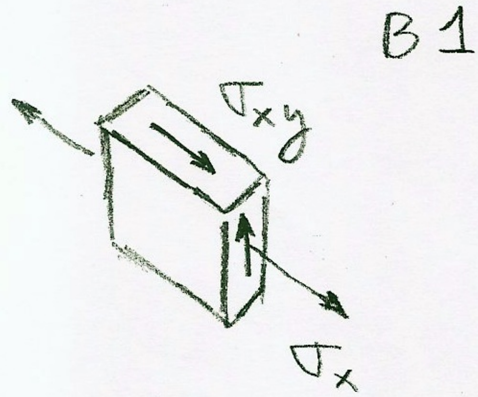
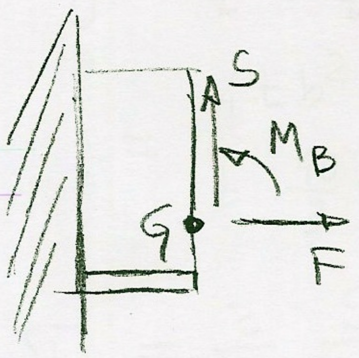
II)











$$M_B = bF + LS$$

$$A = 8tb$$

$$I = (4tb)\,b^2 + \left[\frac{1}{12}\,t\,(4b)^3 + (4tb)\,b^2\right]$$

$$= \left(8 + \frac{16}{3}\right) t b^3$$

$$I = \frac{40}{3}\,t b^3$$

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$$\sigma_x = \frac{F}{A} - \frac{M}{I} b$$

$$= \left(\frac{1}{8tb} - \frac{b^2}{\frac{40}{3} t b^3} \right) F - \frac{bL}{\frac{40}{3} t b^3} S$$

$$= \frac{1}{40} (5-3) \frac{F}{tb} - \frac{3}{40} \frac{L}{tb^2} S$$

$$\sigma_x = \frac{F}{20tb} - \frac{3LS}{40tb^2}$$

$$A^* = 2tb$$

$$Q = (2tb)(2b) = 4tb^2$$

$$\tau_{xy} = \frac{q}{t} = \frac{QS}{tI}$$

$$= \frac{4tb^2 S}{t \, \frac{40}{3} t b^3}$$

$$\tau_{xy} = \frac{3}{10} \frac{S}{tb}$$

$\frac{F}{S}$ $\frac{\cancel{6}^{5} tb^2}{8b * 3L}$