CDS 101/110: Lecture 10-2
Loop Shaping Design Example

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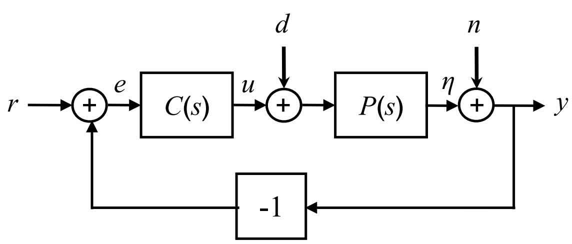
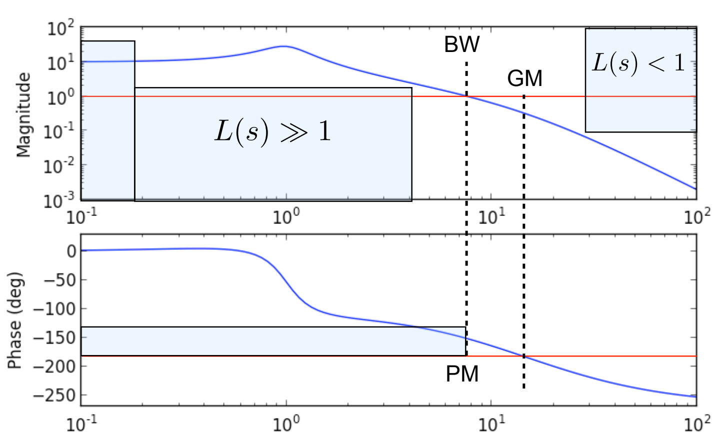
Goals:
- Work through detailed loop shaping-based design

Reading:
- Åström and Murray, Feedback Systems, Sec 12.6

“Loop Shaping”: Design Loop Transfer Function



$$H_{er} = \frac{1}{1+L} \qquad H_{\eta n} = \frac{-L}{1+L}$$



Translate specs to “loop shape”

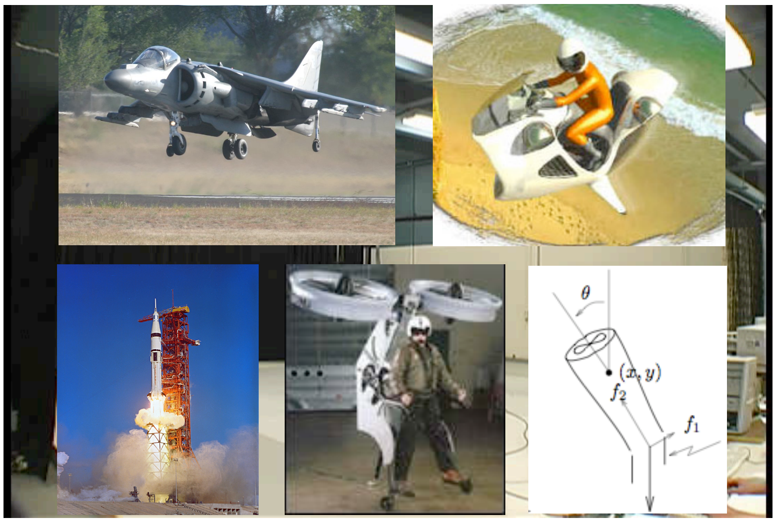
$$L(s) = P(s)C(s)$$

- Design C(s) to obey constraints

Typical loop constraints
- High gain at low frequency
  - Good tracking, disturbance rejection at low freqs
- Low gain at high frequency
  - Avoid amplifying noise
- Sufficiently high bandwidth
  - Good rise/settling time
- Shallow slope at crossover
  - Sufficient phase margin for robustness, low overshoot

Key constraint: slope of gain curve determines phase curve
- Can’t independently adjust
- Eg: slope at crossover sets PM

Example: Control of Vectored Thrust Aircraft



System description

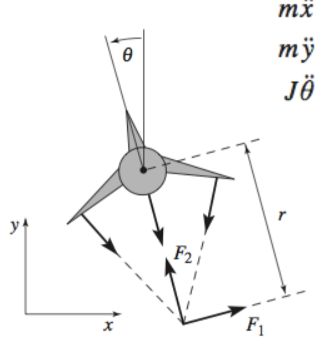
- Vector thrust engine attached to wing
- Inputs: fan thrust, thrust angle (vectored)
- Outputs: position and orientation
- States: x, y, θ + derivatives
- Dynamics: flight aerodynamics

Control approach

- Design "inner loop" control law to regulate pitch (θ) using thrust vectoring
- Second "outer loop" controller regulates the position and altitude by commanding the pitch and thrust
- Basically the same approach as aircraft control laws

Controller structure

Full system dynamics



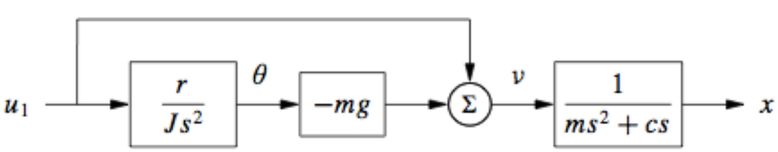
$$m\ddot{x} = -mg\sin\theta - c\dot{x} + u_1\cos\theta - u_2\sin\theta,$$
$$m\ddot{y} = mg(\cos\theta - 1) - c\dot{y} + u_1\sin\theta + u_2\cos\theta,$$
$$J\ddot{\theta} = ru_1.$$

Symbol	Description	Value
m	Vehicle mass	4.0 kg
J	Vehicle inertia, φ_3 axis	0.0475 kg m^2
r	Force moment arm	25.0 cm
c	Damping coefficient	0.05 kg m/s
g	Gravitational constant	9.8 m/s^2

Simplified lateral dynamics (x, θ)

- Linearize the system around hover (equilibrium point)
- Focus on the sideways motion (coupled to roll angle)
- Linearized process dynamics become:

$$H_{\theta u_1} = \frac{r}{Js^2}$$

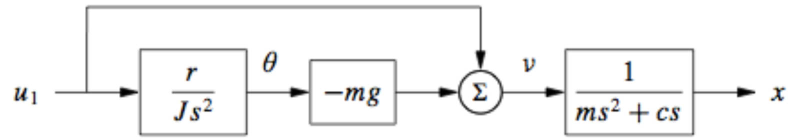


$$H_{xu_1} = \frac{Js^2 - mgr}{Js^2(ms^2 + cs)}$$

Control Strategy: Inner/Outer Loop Design

Control position via roll

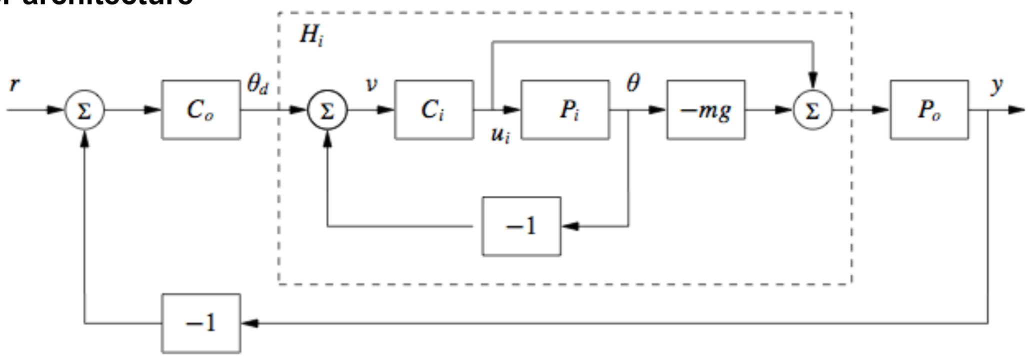
- Use "inner" loop to command u_1 so that θ tracks a desired value



- Use "outer" loop to command θ so that x tracks a desired value
- Motivation: split the design problem into two simpler pieces

$$H_{xu_1} = \frac{Js^2 - mgr}{Js^2(ms^2+cs)}$$

Controller architecture



- Inner loop: design C_i so that roll angle (θ) tracks θ_d
- Outer loop: assume roll angle controller is perfect (H_i = 1) and then design C_o
- Combine inner and outer loop designs to get overall control system design

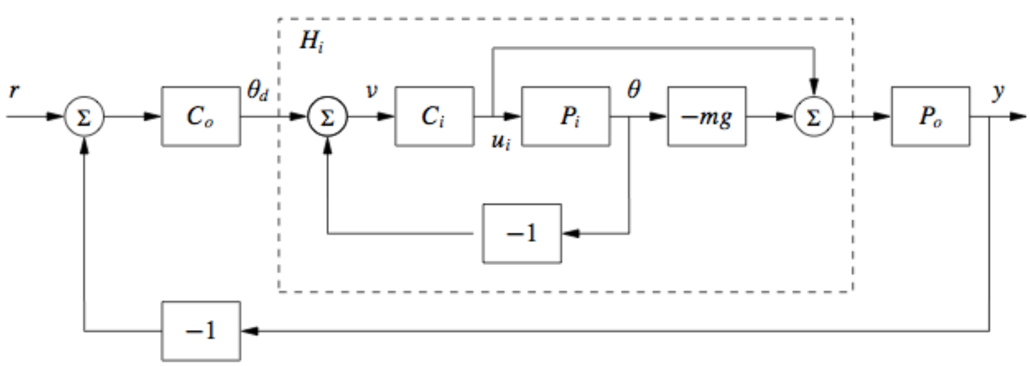
Controller Specification

Overall specification (outer loop)

- Zero steady state error for lateral step response
- Bandwidth of approximately 1 rad/sec
- Phase margin of 45 deg (~20% overshoot)

Inner loop specification: fast tracking of θ_d (so that outer loop can ignore this)

- Set bandwidth to approximately 10X outer loop = 10 rad/sec
- Low frequency error no more than 5%
- Low overshoot (60 deg phase margin should be enough)

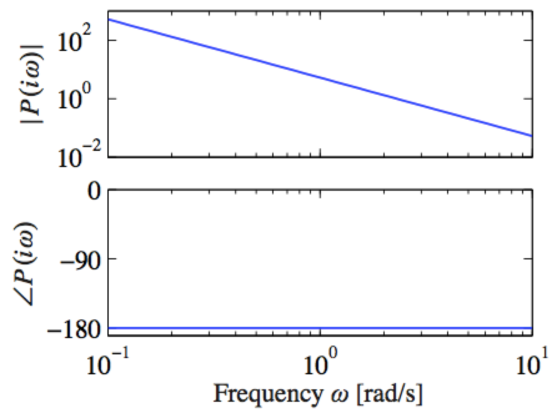


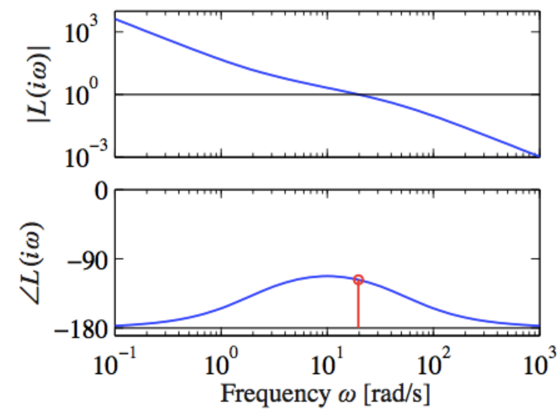
Inner Loop Design

Loop shaping: bandwidth > 10 rad/sec, phase margin > 60 deg

- Process dynamics are second order integrator
- Use lead compensator to add phase; *a* = 2, *b* = 50, *K* = 300
- Get BW = 20 rad/sec, PM = 60 deg

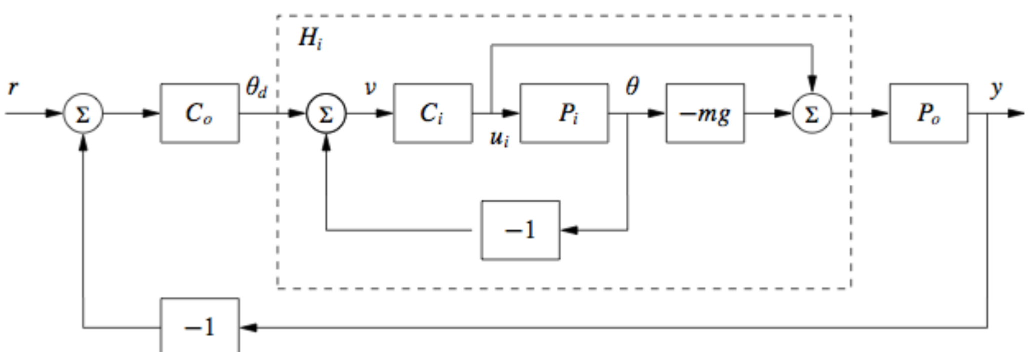
$$H_{\theta u_1} = \frac{r}{Js^2} \qquad C(s) = K\frac{s+a}{s+b}$$



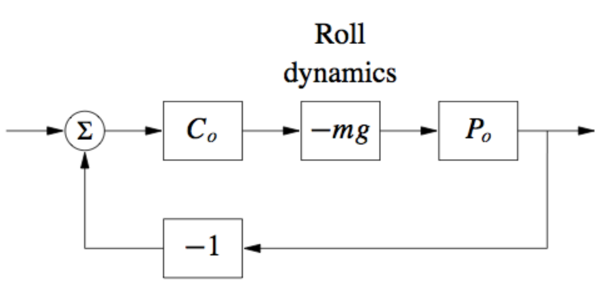


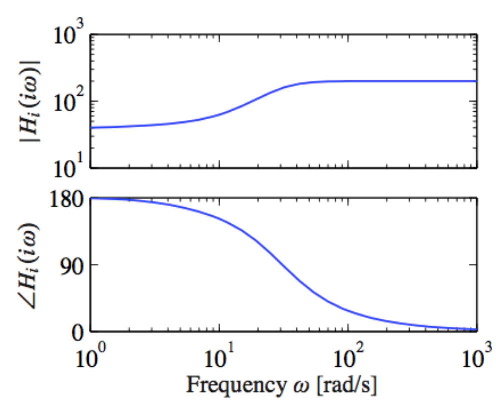
Simplified Inner Loop Dynamics

Full dynamics:



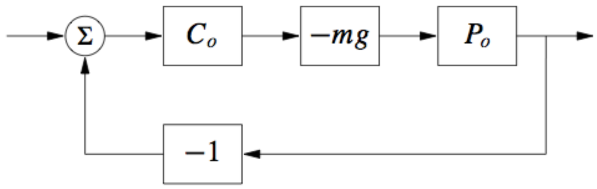
Reduced dynamics:





Outer Loop Design

Design using simplified dynamics



$$P(s) = H_i(0)P_o(s) = \frac{H_i(0)}{ms^2 + cs}$$

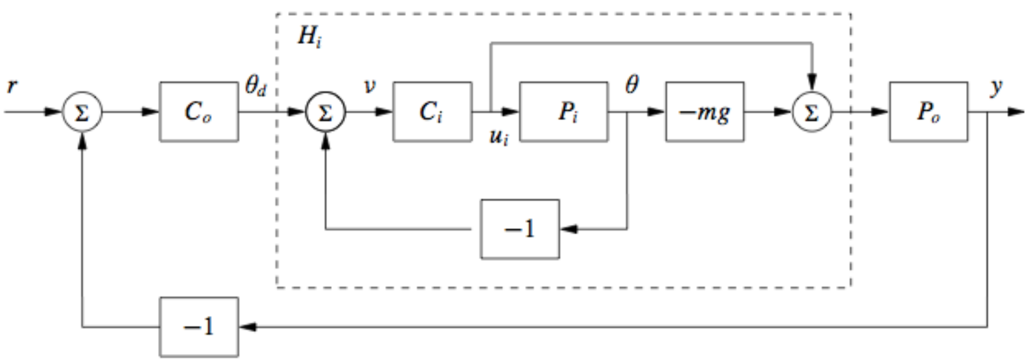
$$C_o(s) = -k_o \frac{s + a_o}{s + b_o}$$

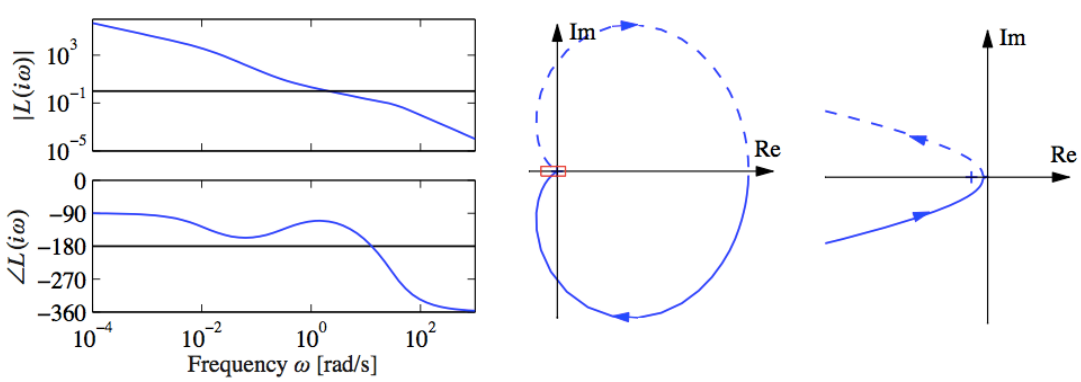
- Process dynamics are (approximately) a double integrator (again!)
- Control design specs
  - Zero steady state error for lateral step response
  - Bandwidth of approximately 1 rad/sec
  - Phase margin of 45 deg (~20% overshoot)
- Can use a lead compensator (again!): put phase lead around 1 rad/sec
- $a_o = 0.3$, $b_o = 10$, $K_i = 2$ => get > 60 deg phase margin, with BW ≈ 1

Remarks

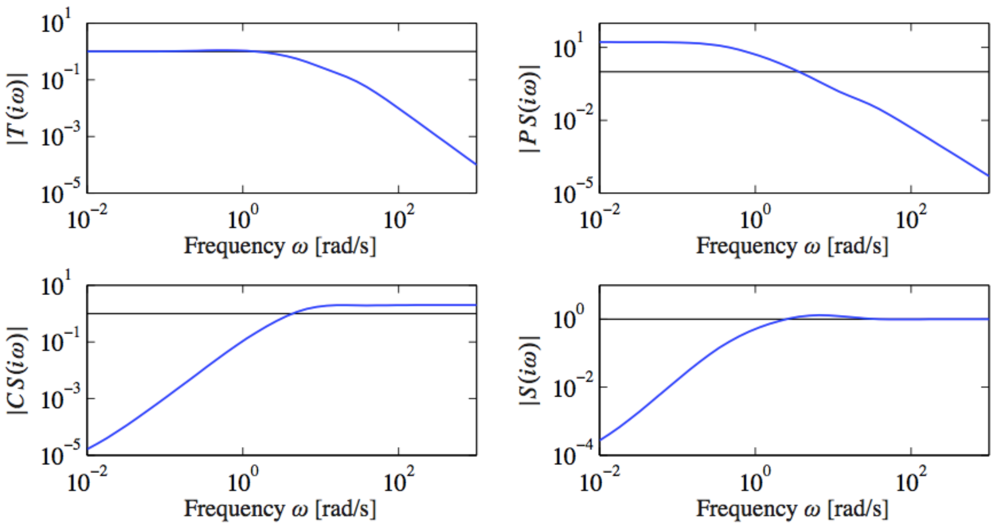
- Note that we will have some residual phase lag from Hi(s) at ω = 1 => set PM = 60 to give a bit of additional margin
- Need to check that the design works with $H_o(s)$ replaced by $H_i(s)$

Final System Design





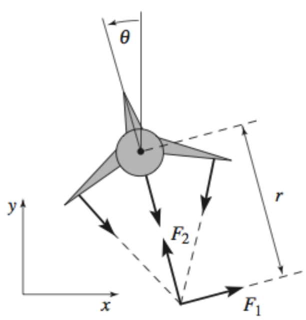
Final Check: Gang of 4



Remarks

- |PS| is a bit large at low frequency => poor disturbance rejection
  - At low frequency C(s) = constant ⇒ P / (1 + PC) → 1/C
- Can fix this by using integral compensation in outer loop controller

Summary



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