Mecanismos de Falhas

Prof. F.A.Rochinha - Outubro de 2020

Ref. e ilustrações : An Introduction to the Mechanics of Solids . Crandall et. All. –Capítulo 5 (ítens: 11, 13, 14 e 15)

https://www.brown.edu/Departments/Engineering/Courses/En1750/Notes/Failure/Failure.htm

Introdução : considerações gerais

- Falhas (não necessariamente catastróficas) e projeto mecânico.

- Falhas “geométricas”: flambagem, estricção ou qualquer mudança que acarrete “perda de funcionalidade”.

- Falhas nos “materiais”: plasticidade, ruptura, fadiga, desgate …

- Teorias (modelos) fenomenológicos (desenvolvidos por experimentos) x princípios fundamentais (atomística por exemplo).

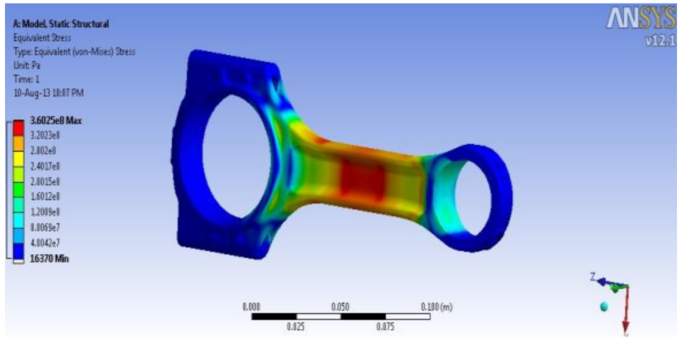
Exemplo 5.1 do Crandall..

Aspectos gerais

- Critérios na forma de equações “simples” tratando de estados de tensão genéricos

$$f(\sigma_x, \sigma_y, \sigma_z, \sigma_{xy}, \ldots) = 0$$

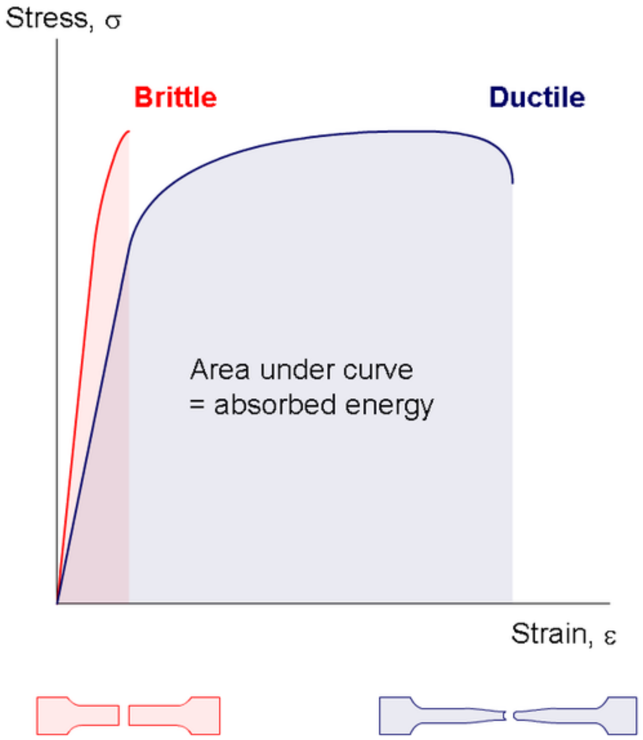
- Identificação de regiões críticas nem sempre trivial (métodos computacionais como Elementos Finitos).



[FE-Analysis of Connecting Rod of I . C . Engine by Using Ansys for Material Optimization](#)
Mr. J. D. Ramani, P. Shukla, D. D. Sharma

Comportamento material

frágil x dúctil



tensões não monotônicas

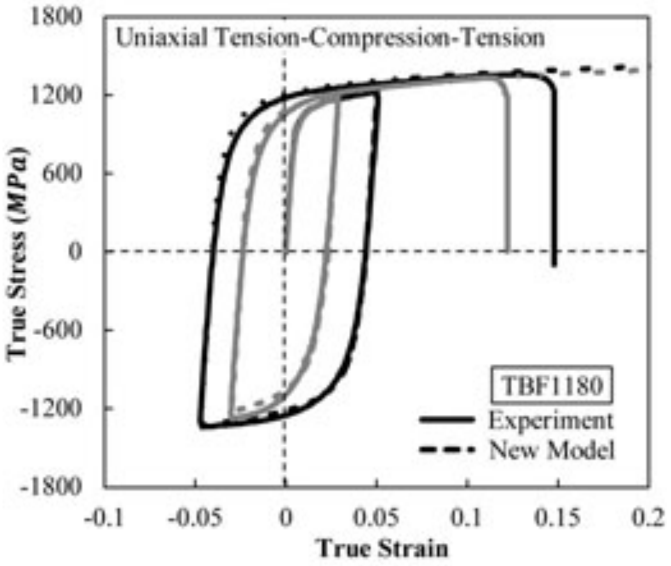


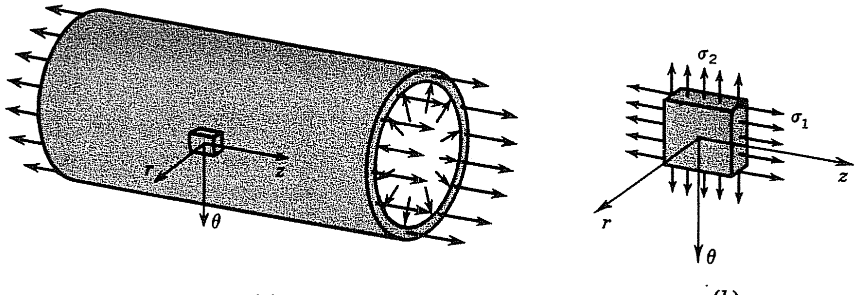
Figure 14. True stress-true strain curves between experimental and modeled results for TBF1180 with a thickness of 1.38mm, under two different tension-compression-tension reversal loading conditions. The uniaxial tensile loading is pursued up to fracture after the reversal loading.

Exemplo 5.1 do Crandall..

Critérios envolvendo plasticidade

Elementos básicos

- Evitar que regiões do corpo plastifiquem.
- Inspiração de uma visão atomística : movimento de discordâncias (reforçando a percepção de que o estado hidrostático de tensões não leva à plastificação).
- Metodologia científica : da observação (experimentação) à teoria.



Fenomenologia para um caso de tensão plana

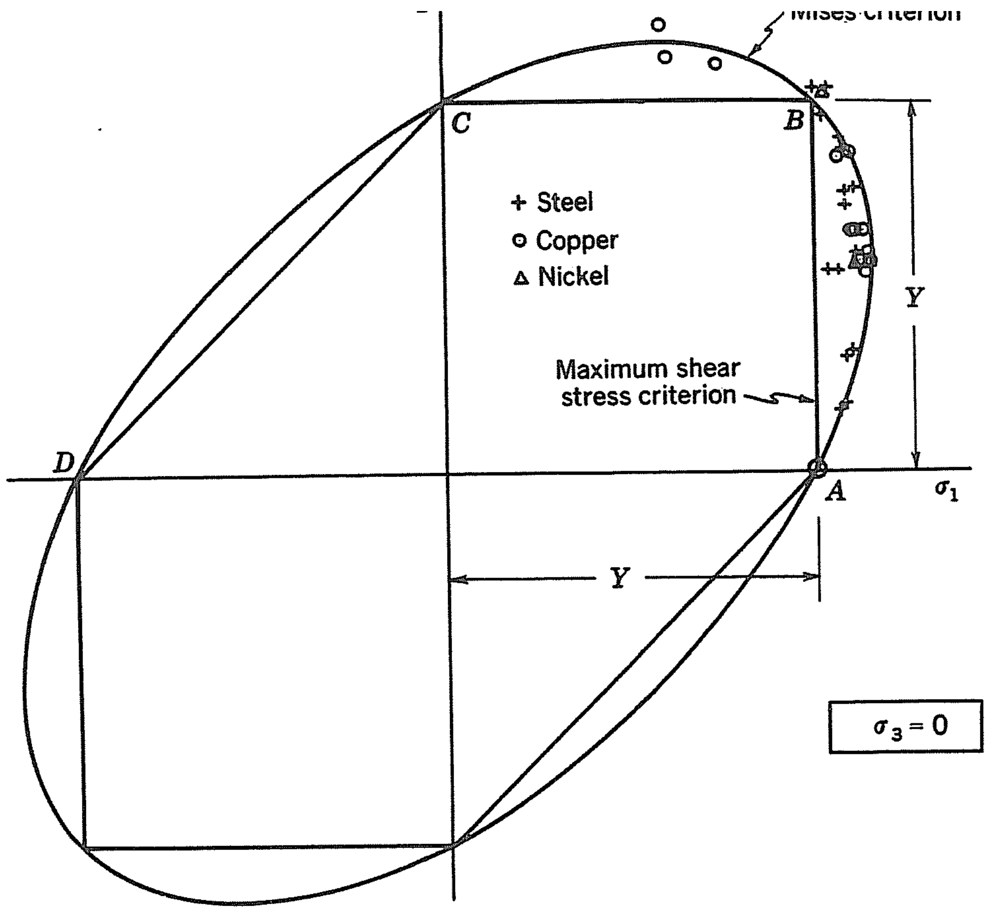


Fig. 5.29 Yielding of thin-walled tubes under combined stress. (*From W. Lode, Versuche uber den Einfluss der mittleren Hauptspannung auf das Fliessen der Metalle Eisen, Kupfer, und Nickel, Z. Physik, vol. 36, pp. 913–939, 1926.*)

Critério de Mises (máxima energia de distorção)

$$\sqrt{\tfrac{1}{3}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}$$

$$= \sqrt{\tfrac{1}{3}[(Y - 0)^2 + (0 - 0)^2 + (0 - Y)^2]} = \sqrt{\tfrac{2}{3}}\, Y$$

$$\sqrt{\tfrac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} = Y$$

$$[\tfrac{1}{2}\{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2\} + 3\tau_{xy}{}^2 + 3\tau_{yz}{}^2 + 3\tau_{zx}{}^2]^{1/2} = Y$$

Y obtido através de um ensaio de tração

Von Mises – máxima energia de distorção

Plastificação acontece quando a máxima energia de distorção é igual à energia de distorção para plastificação em um ensaio de tração.

Tensão Hidrostática

$$\sigma_h = \underbrace{\frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)}_{invariante}$$

Tensão Desviadora

$$\mathbf{S} = \boldsymbol{\sigma} - \sigma_h \mathbf{I}$$

Tensão de Von Mises

$$\sigma_e = \sqrt{\frac{3}{2}\mathbf{S}.\mathbf{S}} = \sqrt{\frac{1}{2}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\sigma_{xy}^2 + \sigma_{xz}^2 + \sigma_{yz}^2)]}$$

Energia de distorção

$$\sqrt{\tfrac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} = Y$$

Critério de Tresca (máxima tensão cisalhante)

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{Y}{2}$$

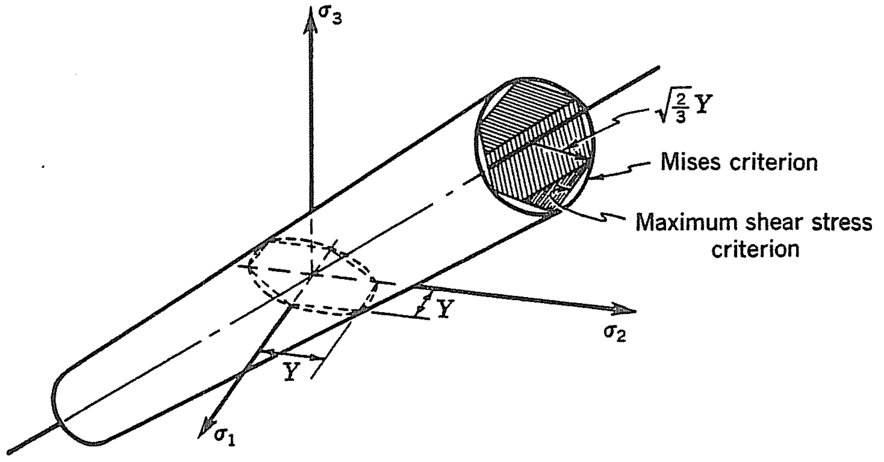
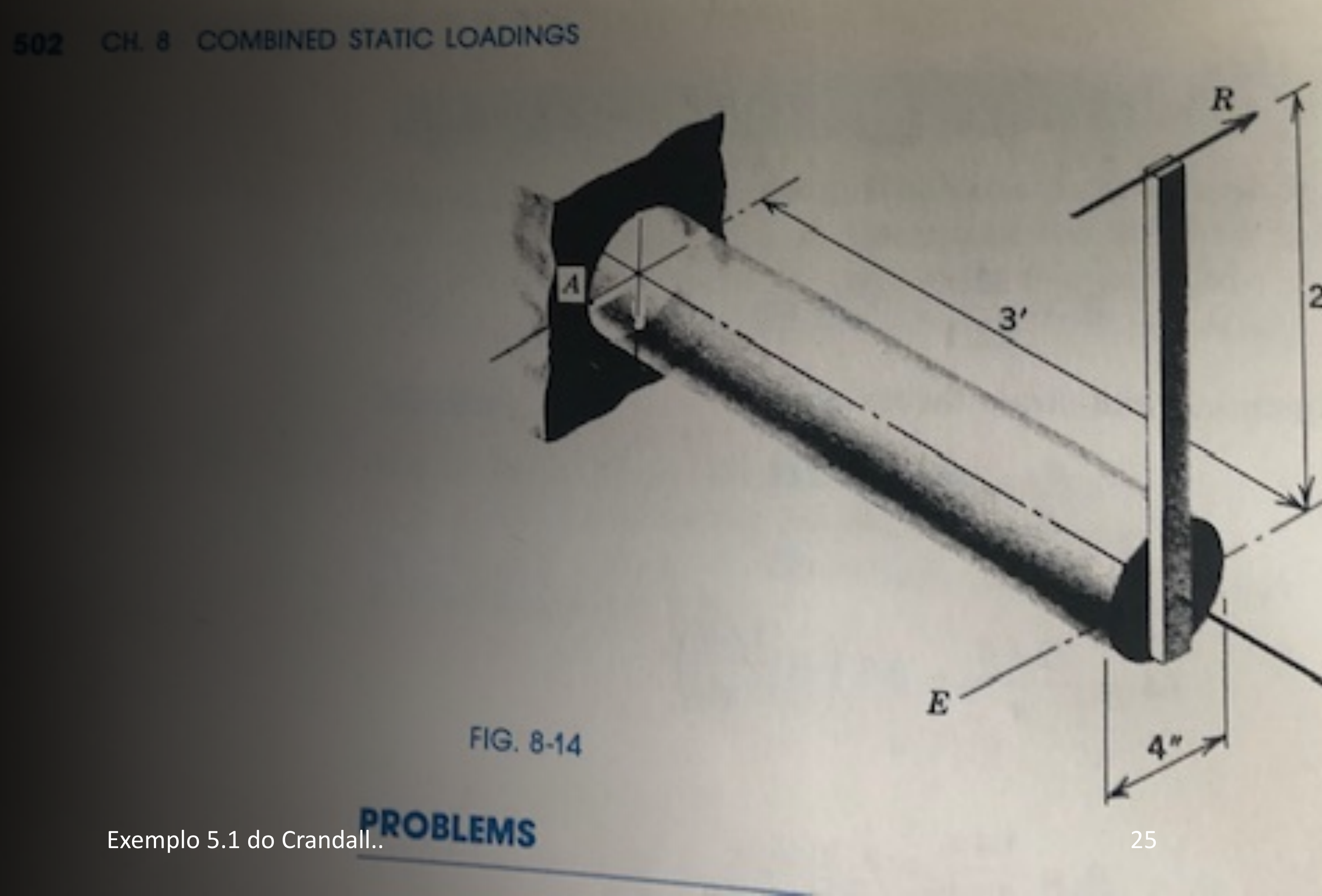


Fig. 5.30 Geometrical representation in principal stress space of the Mises and maximum shear-stress yield criteria.

Exemplo 5.1 do Crandall..

Exemplo – Introduction to Mechanics of Materials. W. Riley, L. Zachary



EXAMPLE 8-7

The solid circular shaft of Fig. 8-14 has a proportional limit of 64 ksi and a Poisson's ratio of 0.30. Determine the value of the load R for failure by slip as predicted by each of the five given theories of failure. Assume that point A is the most severely stressed point.

Solution

When two equal opposite forces R are introduced along line EF of Fig. 8-14, it is evident that the bending moment at point A is $36R$ in.-kips, the torsional moment is $24R$ in.-kips, and the axial load is $12R$ kips. The flexural stress at A is

$$\sigma_1 = \frac{Mc}{I} = \frac{36R(2)}{\pi(2^4)/4} = \frac{18R}{\pi}\ T$$

The direct stress at A is

$$\sigma_2 = \frac{P}{A} = \frac{12R}{\pi(2^2)} = \frac{3R}{\pi}\ T$$

and the sum of the two tensile stresses at A becomes

$$\sigma = \sigma_1 + \sigma_2 = \frac{21R}{\pi}$$

The torsional shearing stress at A is

$$\tau = \frac{Tc}{J} = \frac{24R(2)}{\pi(2^4)/2} = \frac{6R}{\pi}$$

The maximum stresses at A are

$$\tau_{max} = \sqrt{\left(\frac{10.5R}{\pi}\right)^2 + \left(\frac{6R}{\pi}\right)^2} = \frac{12.1R}{\pi}$$

$$\sigma_{p1} = \frac{10.5R}{\pi} + \frac{12.1R}{\pi} = \frac{22.6R}{\pi}\ T$$

and

$$\sigma_{p2} = \frac{10.5R}{\pi} - \frac{12.1R}{\pi} = -\frac{1.6R}{\pi} = \frac{1.6R}{\pi}\ C$$

According to the maximum-normal-stress theory

$$64 = \sigma_f = \sigma_{p1} = 22.6R/\pi$$

from which

$$R = 64\pi/22.6 = \underline{8.90 \text{ kips}}$$ Ans.

According to the maximum-shearing-stress theory

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$$\frac{64}{2} = \tau_f = \tau_{\max} = \frac{12.1R}{\pi}$$

from which

$$R = \frac{32\pi}{12.1} = \underline{8.31 \text{ kips}} \qquad \text{Ans.}$$

The maximum-normal-strain theory gives

$$\frac{\sigma_f}{E} = \frac{\sigma_{p1} - \nu\sigma_{p2}}{E}$$

or

$$64 = \frac{22.6R}{\pi} - 0.3\left(-\frac{1.6R}{\pi}\right)$$

which gives

$$R = \frac{64\pi}{23.1} = \underline{8.70 \text{ kips}} \qquad \text{Ans.}$$

According to the maximum-strain-energy theory

$$\sigma_f^2 = \sigma_{p1}^2 + \sigma_{p2}^2 - 2\nu\sigma_{p1}\sigma_{p2}$$

or

$$64^2 = \left(\frac{22.6R}{\pi}\right)^2 + \left(\frac{-1.6R}{\pi}\right)^2 - 2(0.30)\left(\frac{22.6R}{\pi}\right)\left(\frac{-1.6R}{\pi}\right)$$

which gives

$$R = \frac{64\pi}{23.1} = \underline{8.70 \text{ kips}} \qquad \text{Ans.}$$

According to the maximum-distortion-energy theory

$$\sigma_f^2 = \sigma_{p1}^2 + \sigma_{p2}^2 - \sigma_{p1}\sigma_{p2}$$

or

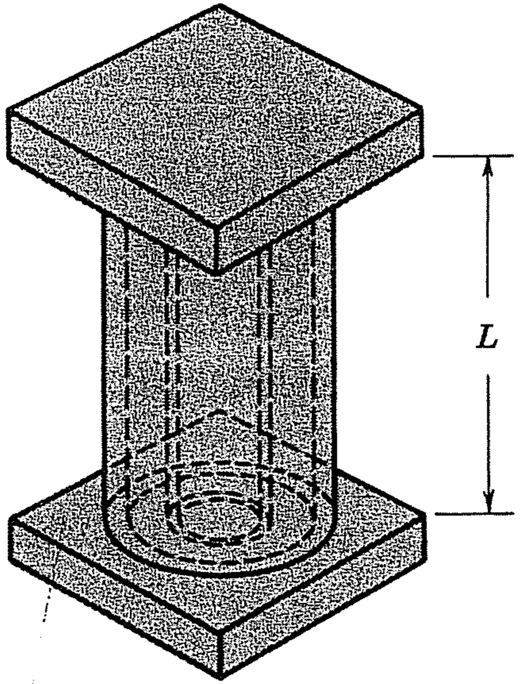
$$64^2 = \left(\frac{22.6R}{\pi}\right)^2 + \left(-\frac{1.6R}{\pi}\right)^2 - \frac{22.6R}{\pi}\left(-\frac{1.6R}{\pi}\right)$$

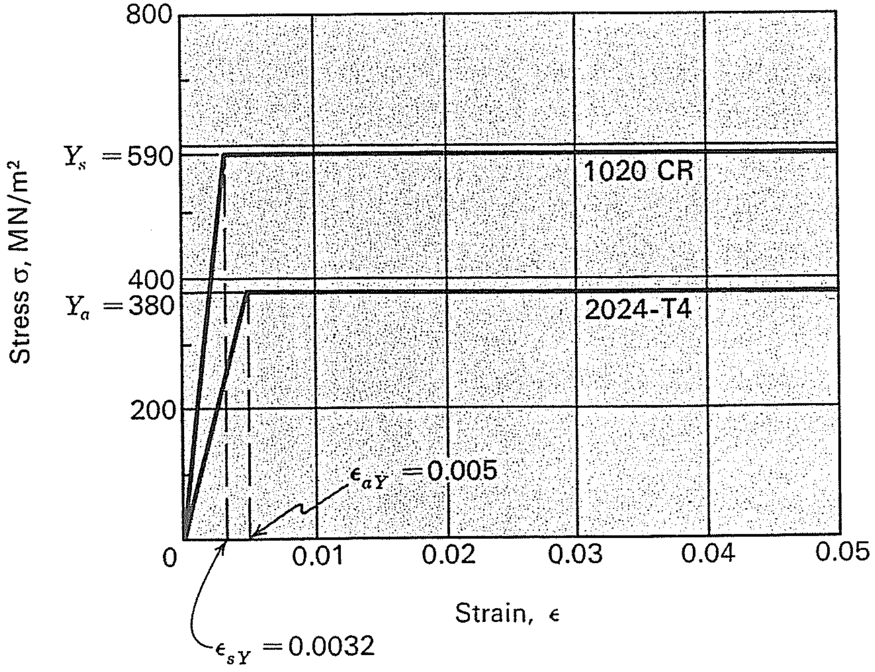
which gives

$$R = \frac{64\pi}{23.4} = \underline{8.59 \text{ kips}} \qquad \text{Ans.}$$

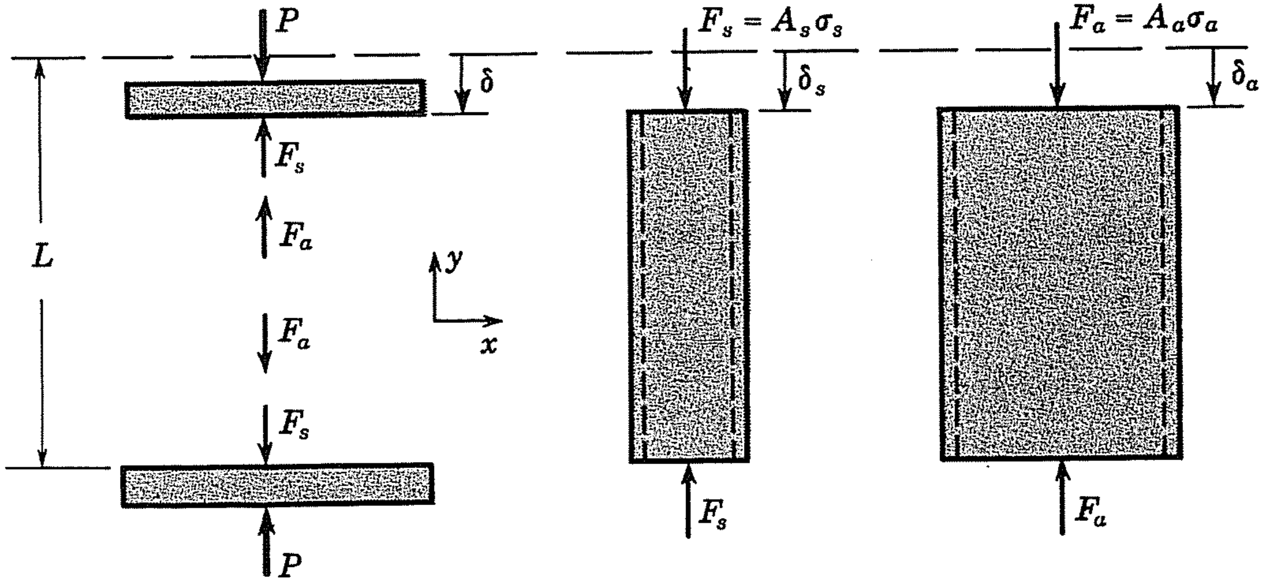
In this example, the maximum-shearing-stress theory is seen to be the most conservative, and the maximum-normal-stress theory gives the least conservative result.

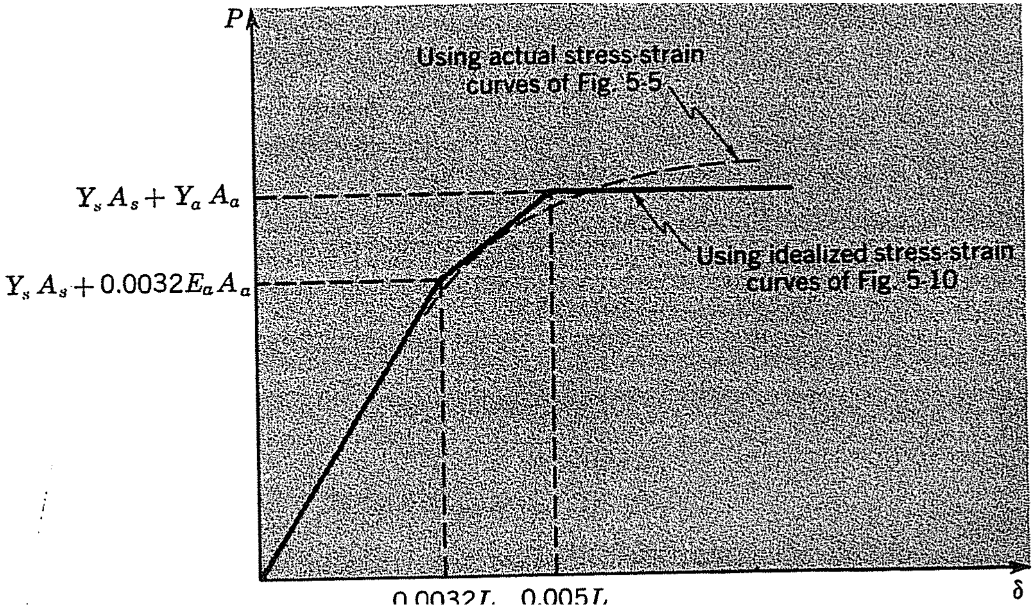
Existe vida depois do limite elástico?





Exemplo 5.1 do Crandall..





Compatibilidade Geométrica

$$\epsilon_s = \epsilon_a = \epsilon = \frac{\delta}{L}$$

Equilíbrio

$$\Sigma F_y = \sigma_s A_s + \sigma_a A_a - P = 0$$

Comportamento constitutivo

$0 \leqq \epsilon \leqq 0.0032$,

$$\sigma_s = E_s\epsilon_s = E_s\epsilon$$
$$\sigma_a = E_a\epsilon_a = E_a\epsilon$$

$0.0032 \leqq \epsilon \leqq 0.00$

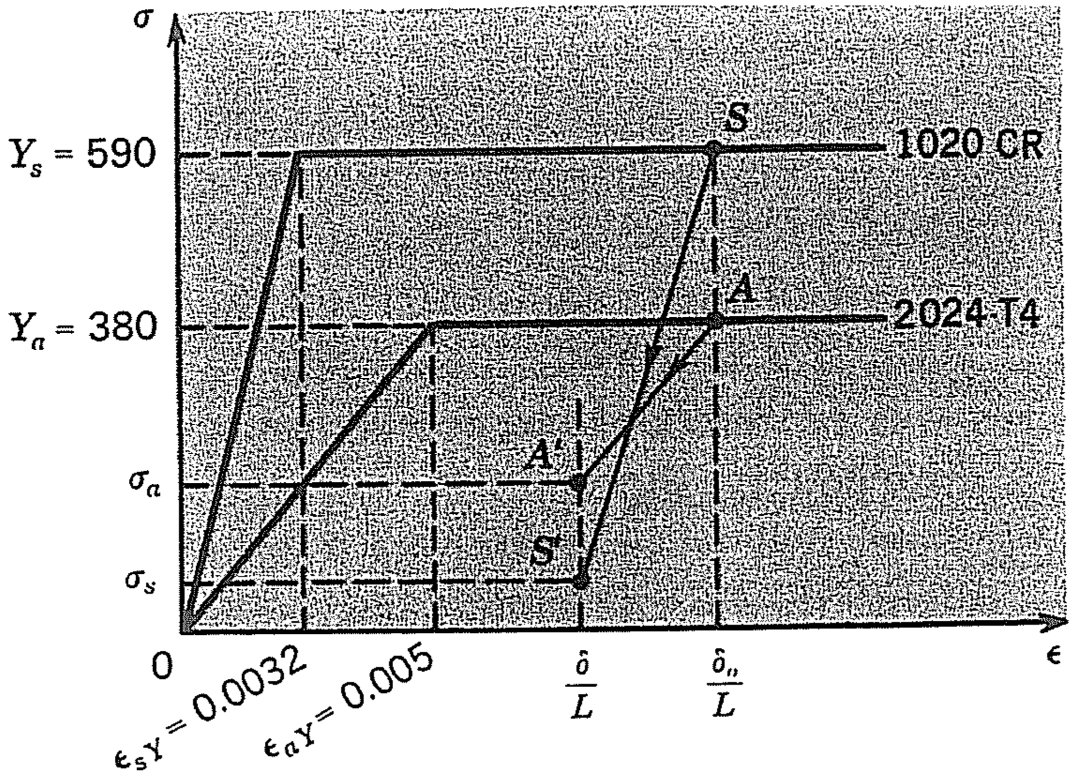
$$\sigma_s = Y_s = 590 \text{ MN/m}^2$$
$$\sigma_a = E_a\epsilon_a = E_a\epsilon$$

$0.005 \leqq \epsilon$

$$\sigma_s = Y_s = 590 \text{ MN/m}^2$$
$$\sigma_a = Y_a = 380 \text{ MN/m}^2$$

carga limite

Descarregando... (tensões residuais)



Equilíbrio

$$\Sigma F_y = \sigma_s A_s + \sigma_a A_a - P = 0$$

Comportamento constitutivo

$$\sigma_s = Y_s - E_s \frac{\delta_o - \delta}{L}$$

$$\sigma_a = Y_a - E_a \frac{\delta_o - \delta}{L}$$

$$\sigma_{s_{residual}} = Y_s \frac{1 - \dfrac{Y_a/E_a}{Y_s/E_s}}{1 + E_s A_s / E_a A_a} = Y_s \frac{1 - \dfrac{\epsilon_{aY}}{\epsilon_{sY}}}{1 + E_s A_s / E_a A_a}$$

$$\sigma_{a_{residual}} = Y_a \frac{1 - \dfrac{Y_s/E_s}{Y_a/E_a}}{1 + E_a A_a / E_s A_s} = Y_a \frac{1 - \dfrac{\epsilon_{sY}}{\epsilon_{aY}}}{1 + E_a A_a / E_s A_s}$$

Exemplo 5.1 do Crandall..