

$$M_2 = R_1 L - \frac{PL}{2}$$

# EQUILÍBRIO (GLOBAL)

$$\begin{aligned} H_1 + H_2 &= 0 \\ R_1 + R_2 - P &= 0 \\ -R_1 L - H_2 L + \frac{PL}{2} + M_2 &= 0 \end{aligned}$$

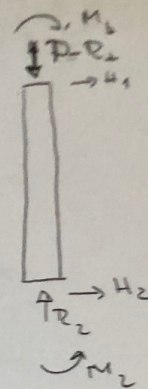
↳ SISTEMA ESTATICAMENTE  
NÃO DETERMINADO!

$$R_1 = \frac{1}{L} [M_2 + \frac{PL}{2}] - H_2$$

# CONDIÇÕES CINEMÁTICAS

$$\left\{ \delta_H = \frac{\delta U}{\delta H_1} = 0 \right.$$

$$\left\{ \delta_V = \frac{\delta U}{\delta R_1} = 0 \right.$$



(2)

$$U = U_{AB} + U_{BC}$$

$$U_{AB} = \underbrace{\int_0^L \frac{H_1^2}{2EA} dx}_{\text{COMPRESSÃO}} + \underbrace{\int_0^L \frac{(P(x - \frac{L}{2}) + R_1 x)^2}{2EI} dx}_{\text{FLEXÃO}}$$

$$U_{BC} = \int_0^L \frac{(R_1 - P)^2}{2EA} dy + \int_0^L \frac{(M_2 + H_1 y)^2}{2EI} dy$$

$$\frac{\partial U}{\partial H_1} = \frac{H_1 L}{EA} + \frac{1}{EI} \int_0^L (M_2 + H_1 y) \left(\frac{y}{2}\right) dy =$$

$$\frac{H_1 L}{EA} + \frac{1}{EI} \left[ M_2 \frac{y^2}{2} \Big|_0^L + H_1 \frac{y^3}{2} \Big|_0^L \right] =$$

$$= \frac{H_1 L}{EA} + \frac{1}{EI} \left[ \frac{R_1 L^3}{2} - \frac{PL^3}{4} + H_1 \frac{L^3}{2} \right] = 0 \quad (i)$$

(3)

$$\frac{\partial U}{\partial R_1} = \frac{1}{EI} \int_0^L (P(x-L/2) + R_1 x) (x) dx + \int_0^L \frac{R_1 - P}{EA} dx$$

$$+ \frac{1}{EI} \int_0^L (M_B + H_1 y) (L) dy$$

$$\frac{\partial U}{\partial R_1} = \frac{1}{EI} \left[ \frac{PL^3}{96} + R_1 \frac{L^3}{3} \right] + \frac{(R_1 - P)L}{EA} +$$

$$+ \frac{1}{EI} \left[ \frac{R_1 L^3}{3} - \frac{PL^3}{3} + \frac{H_1 L^3}{3} \right] = 0 \quad (ii)$$



(4)

Se (i)

$$H_1 = -\frac{1}{EI} \left[ \frac{R_1 L^3}{2} - \frac{PL^3}{4} \right]$$

$$C = \frac{L}{A} + \frac{L^3}{2I}$$

SUBSTITUINDO EM (ii)

$$\frac{1}{I} \left[ \frac{PL^3}{96} + \frac{R_2 L^3}{3} \right] + \frac{R_2 L}{A} - \frac{PL}{A} +$$

$$+ \frac{1}{I} \left[ \frac{R_2 L^3}{3} \right] + \frac{1}{I} \left[ -\frac{PL^3}{3} \right] - \frac{1^3}{3EI^2} \left[ \frac{R_1 L^3}{2} - \frac{PL^3}{4} \right] = 0$$

$$\boxed{R_2 = -\frac{1}{D} \left[ \frac{PL^3}{96I} - \frac{PL}{A} - \frac{PL^3}{3I} - \frac{L^6 P}{12EI^2} \right]}$$

$$D = \frac{2L^3}{3I} + \frac{L}{A} - \frac{L^6}{6EI^2}$$