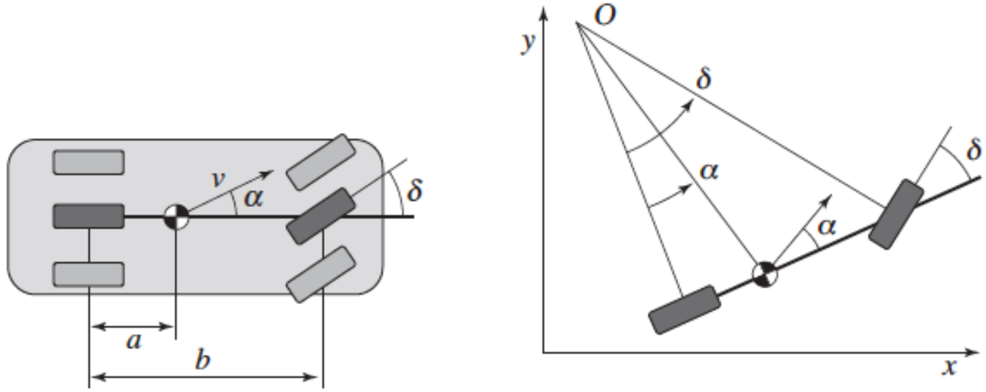
Um exemplo em Controle com Observador

São incluídas passagens do livro dos itens : ex. 3.11, 6.13, 7.4 e 8.3

A visão do controle: da concepção à implementação



(a) Overhead view (b) Bicycle model

Figure 3.17: Vehicle steering dynamics. The left figure shows an overhead view of a vehicle with four wheels. The wheelbase is b and the center of mass at a distance a forward of the rear wheels. By approximating the motion of the front and rear pairs of wheels by a single front wheel and a single rear wheel, we obtain an abstraction called the *bicycle model*, shown on the right. The steering angle is δ and the velocity at the center of mass has the angle α relative the length axis of the vehicle. The position of the vehicle is given by (x, y) and the orientation (heading) by θ.

Construindo um modelo que conecta objetivos e realização

Cinemática do Movimento: movimento do centro e rotação

$$\alpha = \arctan\left(\frac{a \tan \delta}{b}\right)$$

$$\frac{dx}{dt} = v \cos\left(\alpha + \theta\right)$$

$$\frac{dy}{dt} = v \sin\left(\alpha + \theta\right).$$

$$\frac{d\theta}{dt} = \frac{v}{r_c} = \frac{v \sin \alpha}{a} = \frac{v}{a} \sin\left(\arctan\left(\frac{a \tan \delta}{b}\right)\right) \approx \frac{v}{b}\delta,$$

Objetivo do controlador : manter o carro andando em uma linha reta e com velocidade constante

Construindo um modelo que conecta objetivos e realização

$$\frac{d}{dt}\begin{pmatrix} x \\ y \\ \theta \end{pmatrix} = \begin{pmatrix} v\ \cos\left(\alpha(\delta)+\theta\right) \\ v\ \sin\left(\alpha(\delta)+\theta\right) \\ \dfrac{v\ \sin\alpha(\delta)}{a} \end{pmatrix}, \qquad \alpha(\delta) = \arctan\left(\frac{a\tan\delta}{b}\right)$$

Reescrevendo o problema de forma compatível com o objetivo e buscando uma linearização em que o objetivo possa ser traduzido na estabilização entorno de um ponto de equilíbrio...

$$f(x,u) = \begin{pmatrix} v_0 \sin\left(\alpha(u)+x_2\right) \\ \dfrac{v_0 \sin\alpha(u)}{a} \end{pmatrix}, \qquad \alpha(u) = \arctan\left(\frac{a\tan u}{b}\right), \qquad h(x,u) = x_1$$

$$x = (y, \theta)$$

notação

$$u = \delta.$$

Sistema Linearizado e Normalizado

$$A = \left.\frac{\partial f}{\partial x}\right|_{\substack{x=0\\u=0}} = \begin{pmatrix} 0 & v_0 \\ 0 & 0 \end{pmatrix}, \qquad B = \left.\frac{\partial f}{\partial u}\right|_{\substack{x=0\\u=0}} = \begin{pmatrix} a v_0/b \\ v_0/b \end{pmatrix}$$

$$C = \left.\frac{\partial h}{\partial x}\right|_{\substack{x=0\\u=0}} = \begin{pmatrix} 1 & 0 \end{pmatrix}, \qquad D = \left.\frac{\partial h}{\partial u}\right|_{\substack{x=0\\u=0}} = 0,$$

$$z = (x_1/b, x_2) \qquad \tau = v_0 t/b. \qquad \gamma = a/b$$

$$\frac{dz}{d\tau} = \begin{pmatrix} z_2 + \gamma u \\ u \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} z + \begin{pmatrix} \gamma \\ 1 \end{pmatrix} u, \qquad y = \begin{pmatrix} 1 & 0 \end{pmatrix} z,$$

O controlador

$$u = -Kx + k_f r = -k_1 x_1 - k_2 x_2 + k_f r,$$

$$\frac{dx}{dt} = (A - BK)x + Bk_f r = \begin{pmatrix} -\gamma k_1 & 1 - \gamma k_2 \\ -k_1 & -k_2 \end{pmatrix} x + \begin{pmatrix} \gamma k_f \\ k_f \end{pmatrix} r,$$

$$y = Cx + Du = \begin{pmatrix} 1 & 0 \end{pmatrix} x.$$

$$\det(sI - A + BK) = \det \begin{pmatrix} s + \gamma k_1 & \gamma k_2 - 1 \\ k_1 & s + k_2 \end{pmatrix} = s^2 + (\gamma k_1 + k_2)s + k_1.$$

O controlador : calculando os ganhos

$$p(s) = s^2 + 2\zeta_c\omega_c s + \omega_c^2.$$

Polinômio característico determina o comportamento dinâmico do sistema de malha fechada

$$k_1 = \omega_c^2, \qquad k_2 = 2\zeta_c\omega_c - \gamma\omega_c^2.$$

E k_f ?

O papel de K_f

$$u = -Kx + k_f r$$

$$\frac{dx}{dt} = (A - BK)x + Bk_f r.$$

$$x_e = -(A - BK)^{-1} B k_f r, \qquad y_e = C x_e$$

$$y_e = r \qquad k_f = -1/\left(C(A - BK)^{-1} B\right)$$

Como escolher o ganho do controlador?

$$k_{\mathrm{f}} = k_1 = \omega_{\mathrm{c}}^2,$$

$$u = k_1(r - x_1) - k_2 x_2 = \omega_{\mathrm{c}}^2(r - x_1) - (2\zeta_{\mathrm{c}}\omega_{\mathrm{c}} - \gamma\omega_{\mathrm{c}}^2)x_2.$$

Uma breve excursão sobre sistemas dinâmicos lineares de segunda ordem : estabilidade

$$\ddot{q} + 2\zeta\omega_0\dot{q} + \omega_0^2 q = k\omega_0^2 u, \qquad y = q.$$

Um simples "massa-mola"

autovalores

$$\frac{dx}{dt} = \begin{pmatrix} 0 & \omega_0 \\ -\omega_0 & -2\zeta\omega_0 \end{pmatrix} x + \begin{pmatrix} 0 \\ k\omega_0 \end{pmatrix} u, \qquad y = \begin{pmatrix} 1 & 0 \end{pmatrix} x, \quad x = (q, \dot{q}/\omega_0) \qquad \lambda = -\zeta\omega_0 \pm \omega_0\sqrt{(\zeta^2 - 1)},$$

$\zeta > 1$

$$y(t) = \frac{\beta x_{10} + x_{20}}{\beta - \alpha} e^{-\alpha t} - \frac{\alpha x_{10} + x_{20}}{\beta - \alpha} e^{-\beta t}, \qquad \alpha = \omega_0(\zeta + \sqrt{\zeta^2 - 1}) \quad \beta = \omega_0(\zeta - \sqrt{\zeta^2 - 1})$$

$\zeta = 1$

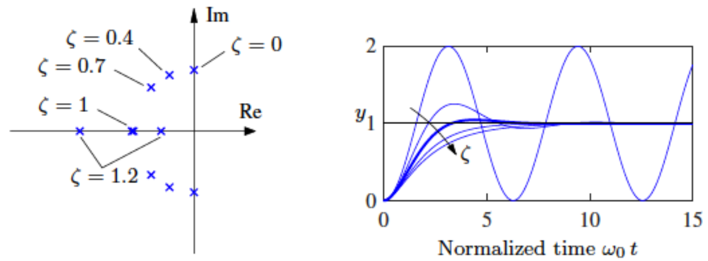
$$y(t) = e^{-\zeta\omega_0 t}(x_{10} + (x_{20} + \zeta\omega_0 x_{10})t)$$

$0 < \zeta < 1$

$$y(t) = e^{-\zeta\omega_0 t}\left(x_{10}\cos\omega_d t + \left(\frac{\zeta\omega_0}{\omega_d} x_{10} + \frac{1}{\omega_d} x_{20}\right)\sin\omega_d t\right) \qquad \omega_d = \omega_0\sqrt{1 - \zeta^2}$$

Uma breve excursão sobre sistemas dinâmicos lineares de segunda ordem : resposta a um degrau

$$
y(t) = \begin{cases}
k\left(1 - e^{-\zeta\omega_0 t}\cos\omega_{\mathrm{d}} t - \dfrac{\zeta}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_0 t}\sin\omega_{\mathrm{d}} t\right), & \text{if } \zeta < 1; \\
k\left(1 - e^{-\omega_0 t}(1+\omega_0 t)\right), & \text{if } \zeta = 1; \\
k\left(1 - \frac{1}{2}\left(\frac{\zeta}{\sqrt{\zeta^2-1}} + 1\right)e^{-\omega_0 t(\zeta - \sqrt{\zeta^2-1})} \right. & \\
\qquad \left. + \frac{1}{2}\left(\frac{\zeta}{\sqrt{\zeta^2-1}} - 1\right)e^{-\omega_0 t(\zeta + \sqrt{\zeta^2-1})}\right), & \text{if } \zeta > 1,
\end{cases}
$$

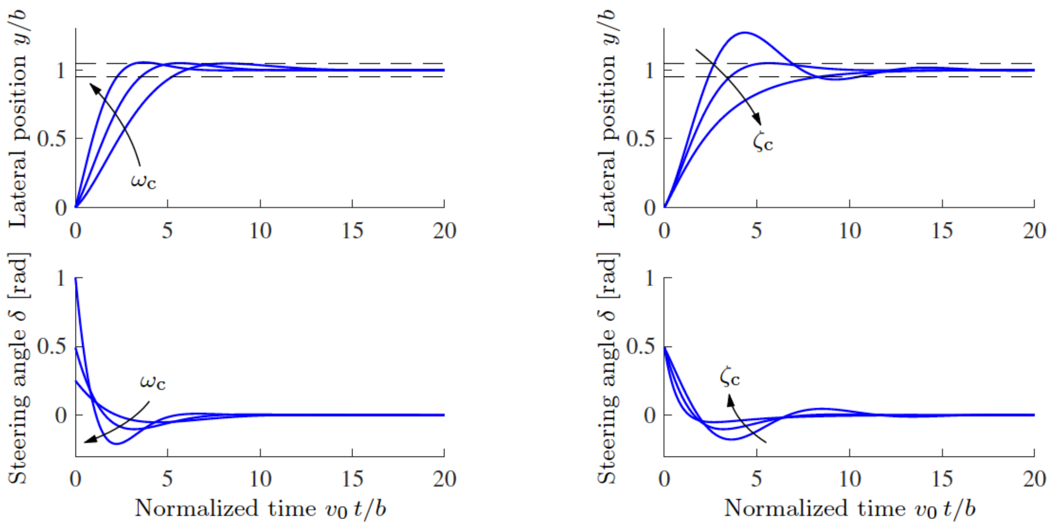


(a) Eigenvalues (b) Step responses

Figure 7.8: Step response for a second-order system. Normalized step responses for the system (7.23) for $\zeta = 0$, 0.4, 0.7 (thicker), 1, and 1.2. As the damping ratio is increased, the rise time of the system gets longer, but there is less overshoot. The horizontal axis is in scaled units $\omega_0 t$; higher values of ω_0 result in a faster response (rise time and settling time).

Combinando estabilização e desempenho para projetar o controlador

Resposta do Sistema de Malha Fechada



(a) Unit step response for varying ω_c

(b) Unit step response for varying ζ_c

Figure 7.6: State feedback control of a steering system. Unit step responses (from zero initial condition) obtained with controllers designed with $\zeta_c = 0.7$ and $\omega_c = 0.5$, 0.7, and 1 [rad/s] are shown in (a). The dashed lines indicate $\pm 5\%$ deviations from the setpoint. Notice that response speed increases with increasing ω_c, but that large ω_c also give large initial control actions. Unit step responses obtained with a controller designed with $\omega_c = 0.7$ and $\zeta_c = 0.5$, 0.7, and 1 are shown in (b).

Agora usando um observador – princípio da separação (controlador – observador)

$$\frac{dx}{dt} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x + \begin{pmatrix} \gamma \\ 1 \end{pmatrix} u, \qquad y = \begin{pmatrix} 1 & 0 \end{pmatrix} x.$$

$$W_o = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Matriz de observabilidade

$$A - LC = \begin{pmatrix} -l_1 & 1 \\ -l_2 & 0 \end{pmatrix}$$

Projetando o (Ganho) do Observador

$$\det(sI - A + LC) = \det\begin{pmatrix} s + l_1 & -1 \\ l_2 & s \end{pmatrix} = s^2 + l_1 s + l_2.$$

$$s^2 + p_1 s + p_2 = s^2 + 2\zeta_o\omega_o s + \omega_o^2$$

$$l_1 = p_1 = 2\zeta_o\omega_o, \qquad l_2 = p_2 = \omega_o^2$$

L_2 regula a frequência de oscilação entorno da trajetória real e l1 determina o tempo para alcançar essa trajetória

$$\frac{d\hat{x}}{dt} = A\hat{x} + Bu + L(y - C\hat{x}) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \hat{x} + \begin{pmatrix} \gamma \\ 1 \end{pmatrix} u + \begin{pmatrix} l_1 \\ l_2 \end{pmatrix} (y - \hat{x}_1)$$

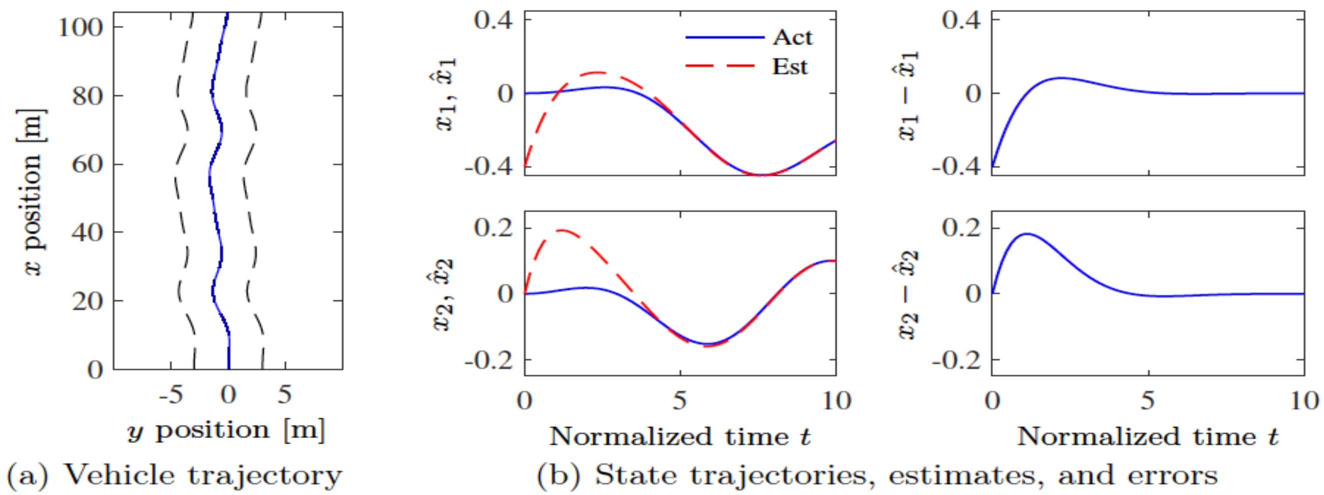


Figure 8.6: Simulation of an observer for a vehicle driving on a curvy road. (a) The vehicle trajectory, as viewed from above, with the lane boundaries shown as dashed lines. (b) The response of the observer with an initial position error. The plots on the left show the lateral deviation x_1 and the lateral velocity x_2 with solid lines and their estimates $\hat{x}_1$ and $\hat{x}_2$ with dashed lines. The plots on the right show the estimation errors. The parameters used to design the estimator were $\omega_o = 1$ and $\zeta_o = 0.7$.