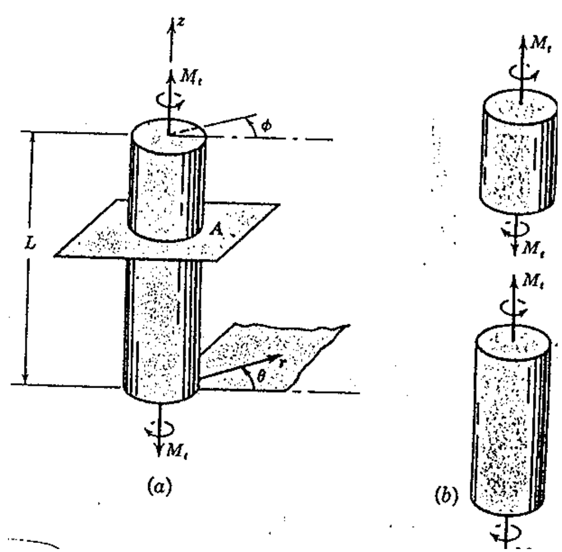
Distribuição de Tensões e Deformações em um Eixo Cilíndrico em Torção

$$\sigma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \tau_{\theta z} \\ 0 & \tau_{\theta z} & 0 \end{bmatrix}$$

$$\epsilon = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon_{\theta z} \\ 0 & \epsilon_{\theta z} & 0 \end{bmatrix}$$

$$\tau_{\theta z} = \frac{Tr}{J}$$



$$\gamma_{\theta z} = \frac{Tr}{GJ} = r\frac{d\phi}{dz}$$

$$\frac{d\phi}{dz} = \frac{T}{GJ}$$

Exemplos Crandall et all.

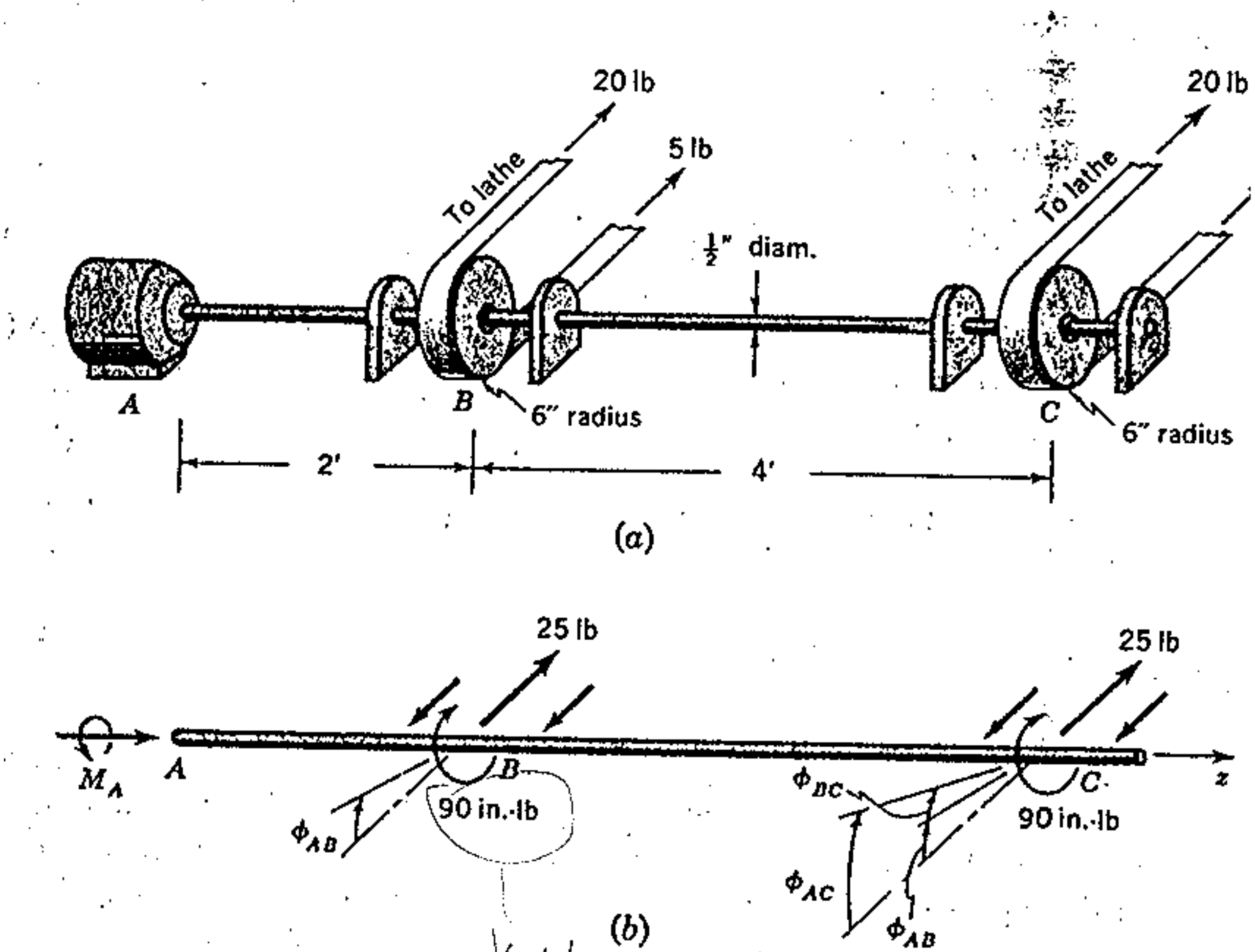
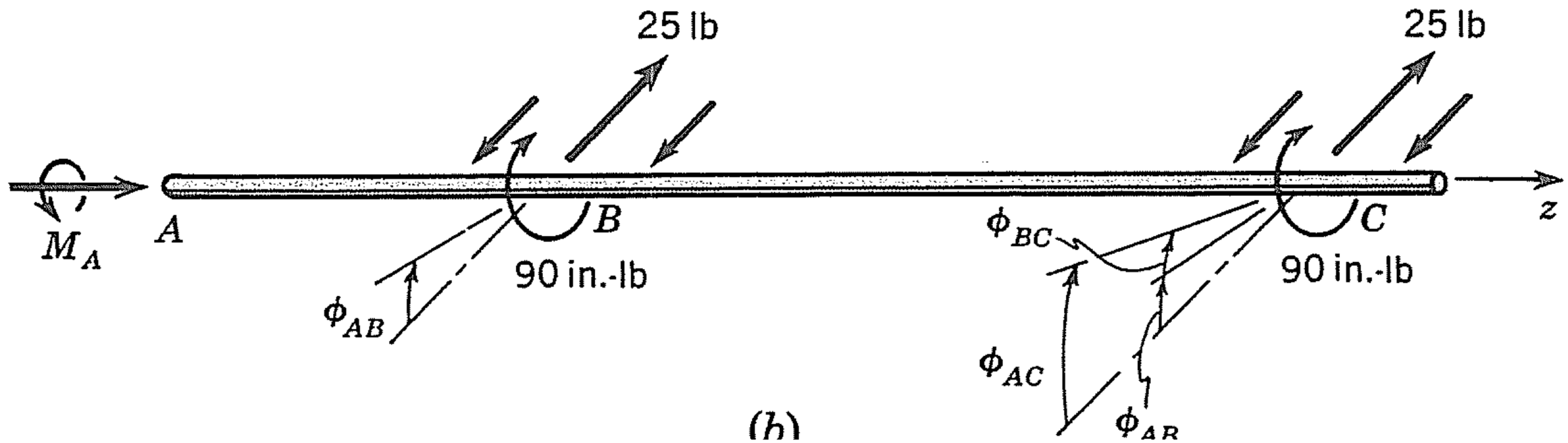


Fig. 6.10 Example 6.1.



(b)

EQUILIBRIUM

Establishing moment equilibrium, we have, since all moment vectors are parallel to z, $\Sigma \mathbf{M}_A = 0$ if

$$M_A - 90 - 90 = 0$$
$$M_A = 180 \text{ in.-lb} \tag{a}$$

The twisting moments in sections AB and BC of the shaft are then clearly

$$M_{AB} = 180 \text{ in.-lb} \qquad M_{BC} = 90 \text{ in.-lb} \tag{b}$$

GEOMETRIC COMPATIBILITY

We wish to find the angle ϕ_{AC} which describes the rotation of the end C with respect to the end A. From the sketch in Fig. 6.10b we see that

$$\phi_{AC} = \phi_{AB} + \phi_{BC} \tag{c}$$

LOAD-DEFORMATION RELATION

From Eq. (6.8) we have

$$\phi_{AB} = \frac{M_{AB}L_{AB}}{GI_z} \qquad \phi_{BC} = \frac{M_{BC}L_{BC}}{GI_z} \tag{d}$$

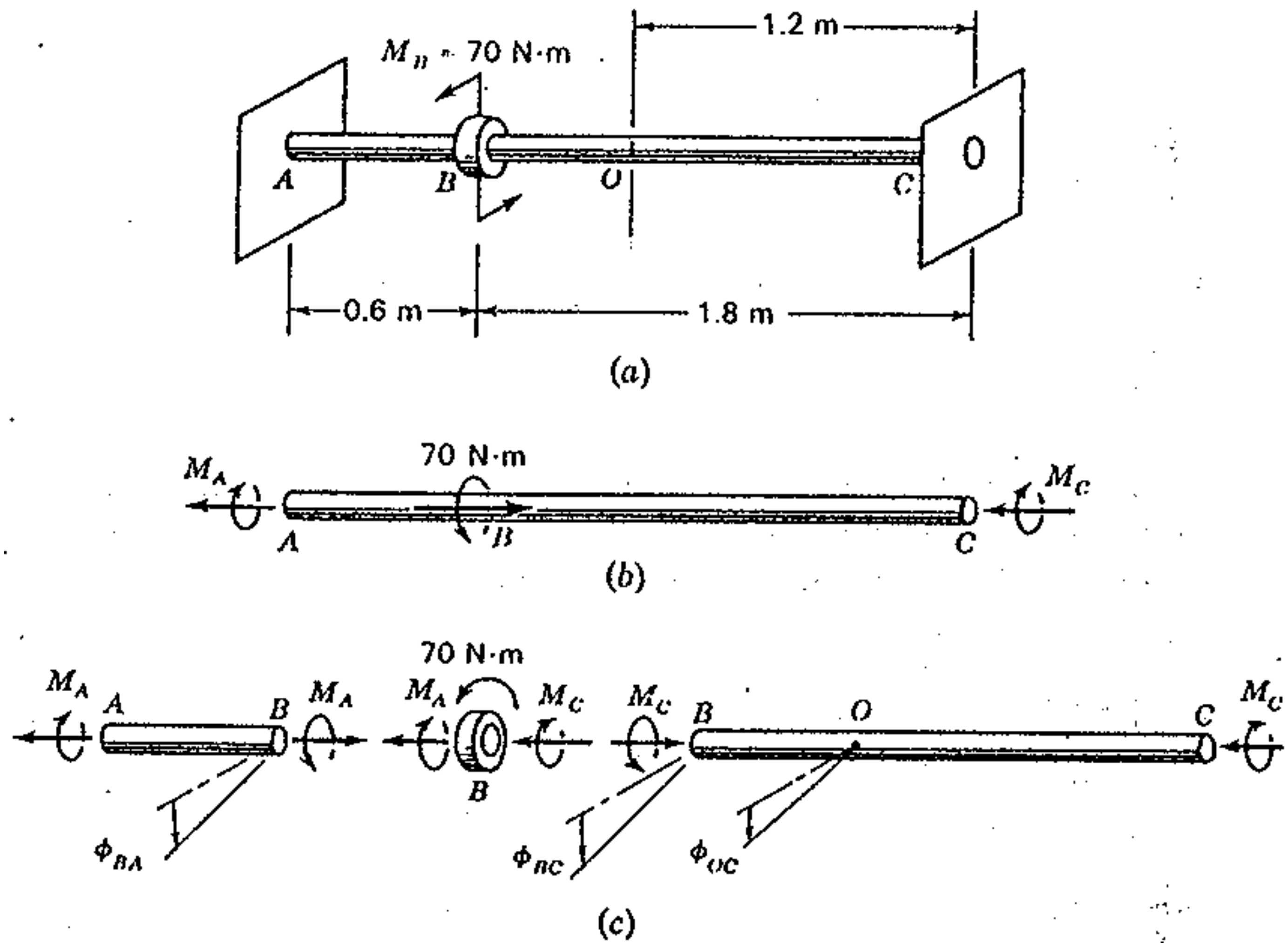
where, from Table 5.1, $G = 11.5 \times 10^6$ psi. Combining Eqs. (b), (c), and (d), and having proper regard for units, we find

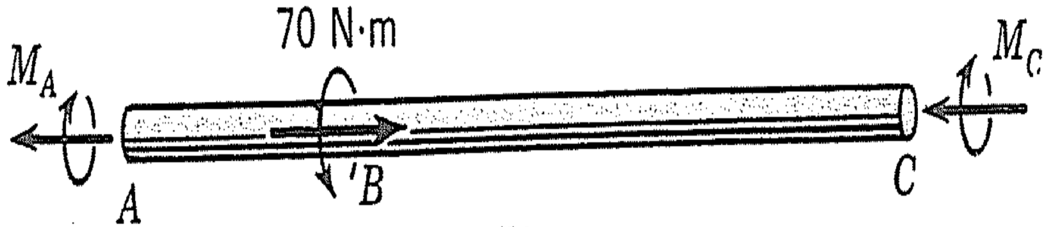
$$\phi_{AC} = 0.123 \text{ rad} = 7.0^\circ \tag{e}$$

The maximum shear stress occurs at the outside of the shaft in section AB. Using (6.9) we find

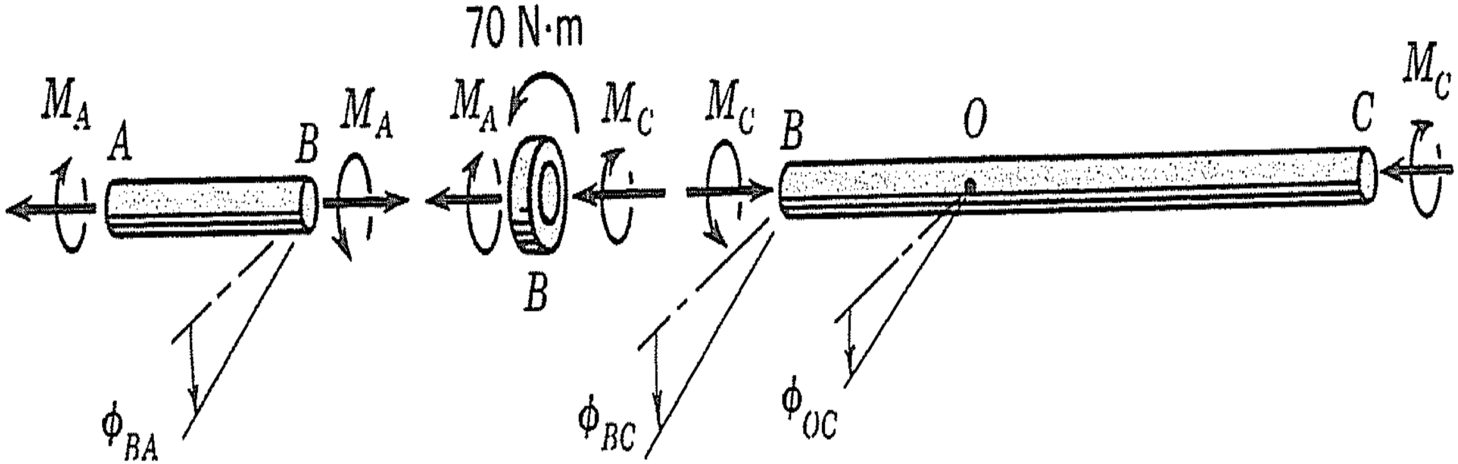
$$(\tau_{\theta z})_{\max} = \frac{M_{AB}r_0}{I_z} = 7{,}300 \text{ psi} \tag{f}$$

Indeterminado estaticamente





(b)



(c)

EQUILIBRIUM

Satisfying moment equilibrium for the complete shaft shown in Fig. 6.11*b* (or for the middle segment in Fig. 6.11*c*) yields

$$M_A + M_C - 70 = 0 \tag{a}$$

GEOMETRIC COMPATIBILITY

Continuity of the shaft at the point *B* requires that

$$\phi_{BC} = \phi_{BA} \tag{b}$$

LOAD-DEFORMATION RELATION

From Eq. (6.8) we have

$$\phi_{BA} = \frac{M_A L_{AB}}{GI_z} \qquad \phi_{BC} = \frac{M_C L_{BC}}{GI_z}$$

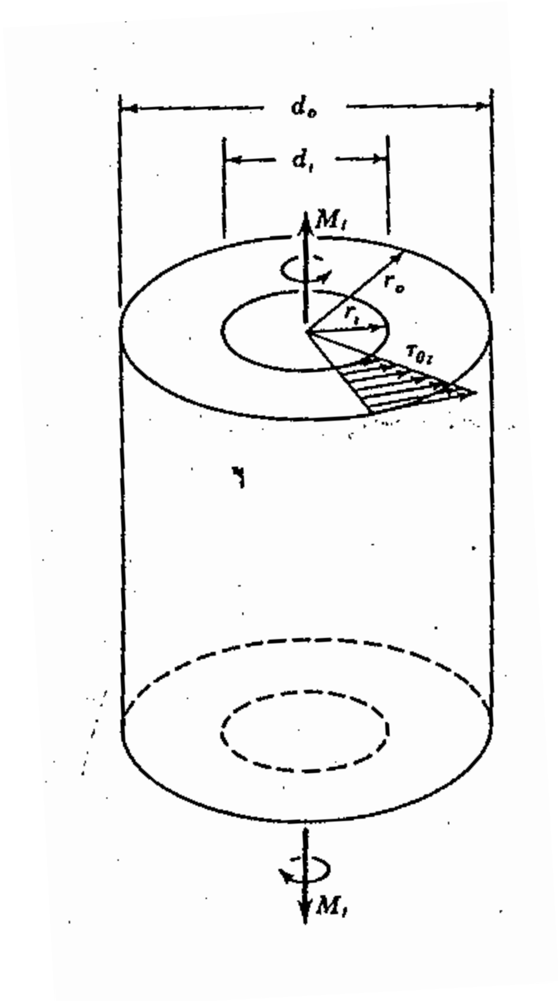
$$\phi_{OC} = \frac{M_C L_{OC}}{GI_z} \tag{c}$$

$$M_A = \frac{L_{BC}}{L_{AC}} M_B = 52.5 \text{ N·m}$$

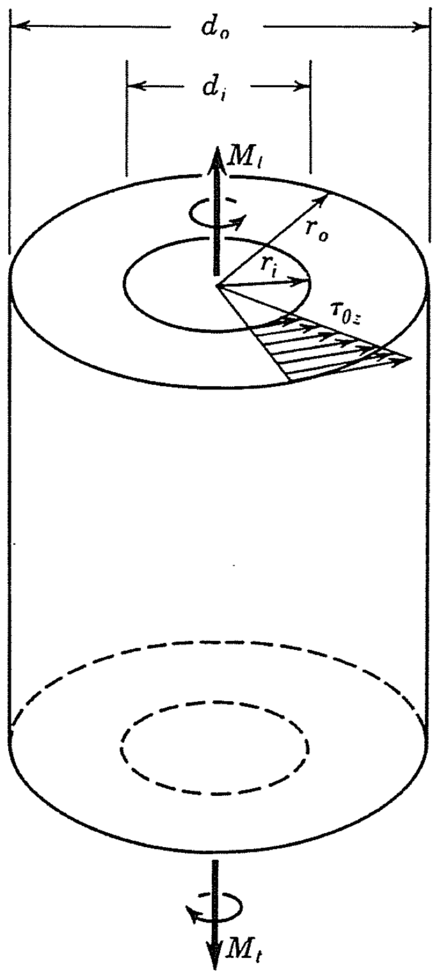
$$M_C = \frac{L_{AB}}{L_{AC}} M_B = 17.5 \text{ N·m}$$

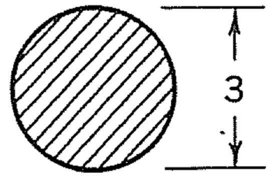
$$\phi_{OC} = 0.021 \text{ rad} = 1.20°$$

Eixo Vazado



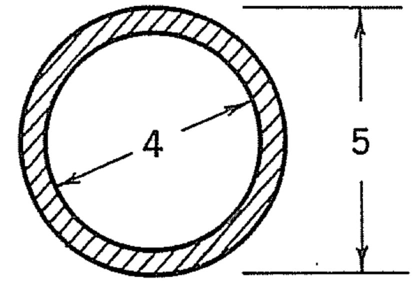
O que muda???





Shaft 1

$$(\tau_{\theta z})_{\max.} = \tau_1$$
$$\frac{M_t}{\phi} = k_1$$



Shaft 2

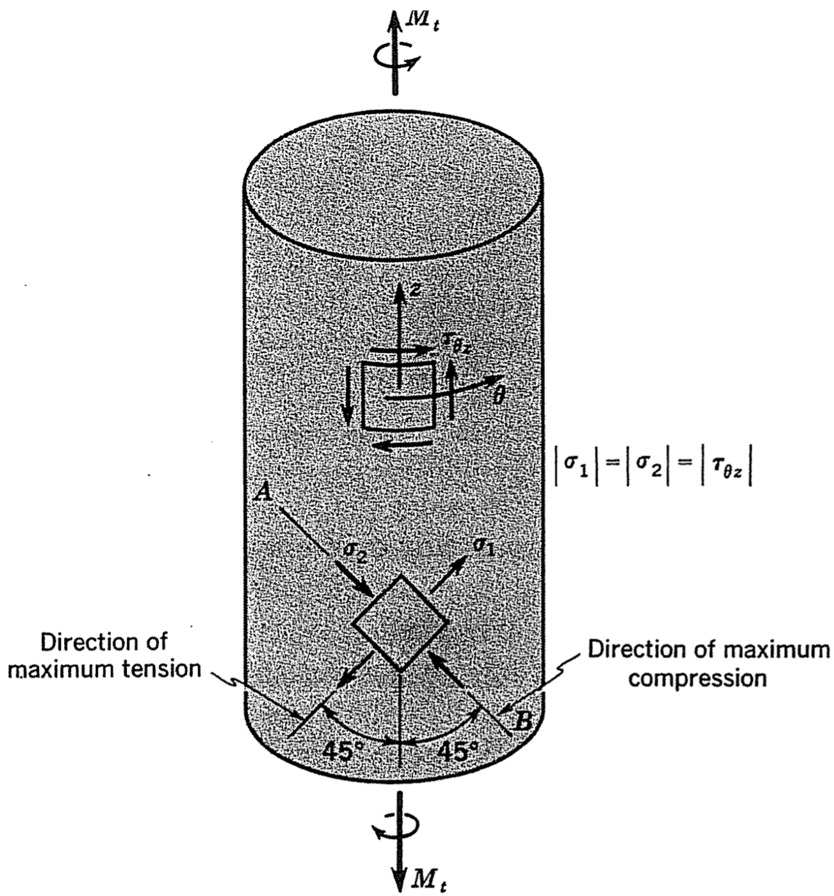
$$(\tau_{\theta z})_{\max.} = \tau_2$$
$$\frac{M_t}{\phi} = k_2$$

When both shafts are twisted by the same twisting moment,

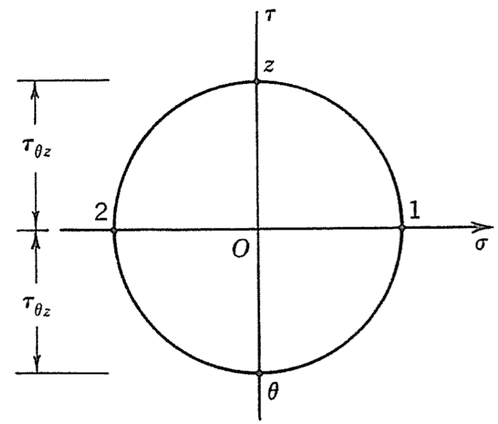
$$\text{Stress ratio } = \frac{\tau_2}{\tau_1} = \frac{15}{41} = 0.37$$

$$\text{Stiffness ratio } = \frac{k_2}{k_1} = \frac{41}{9} = 4.56$$

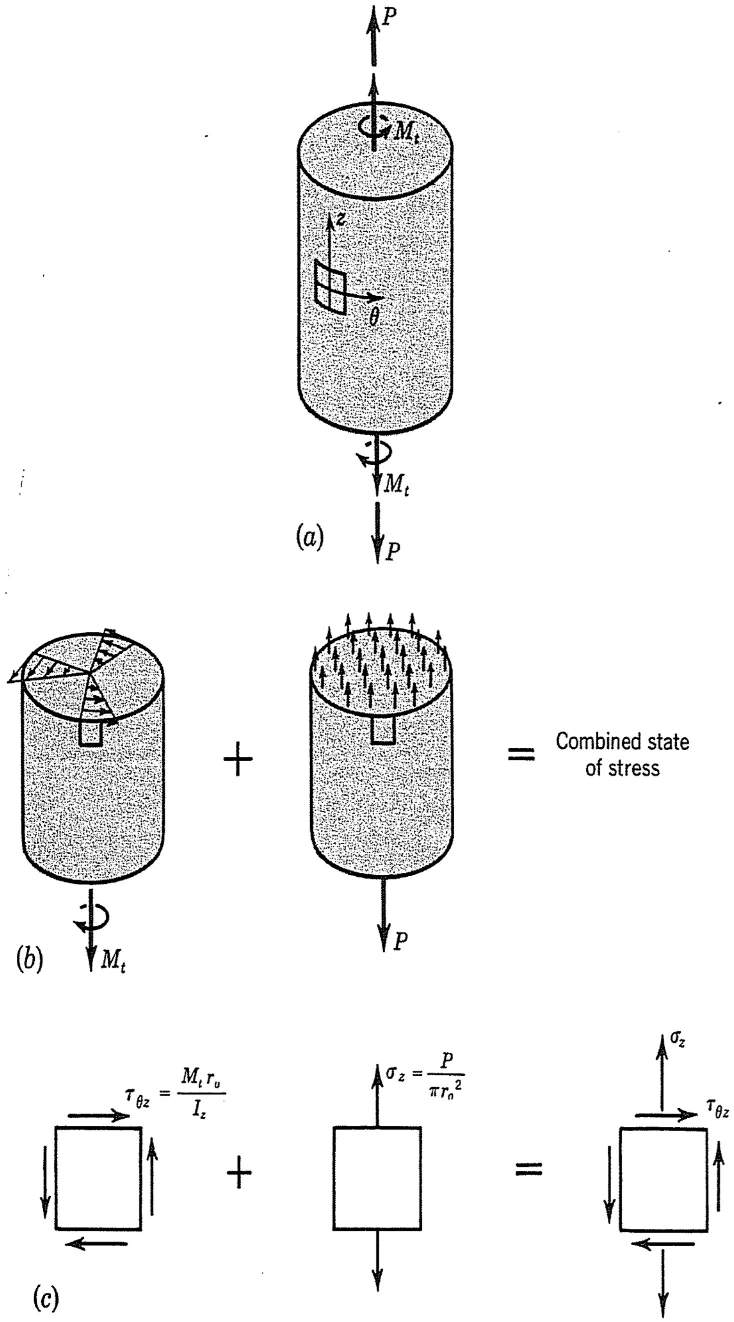
$$I_z = \frac{\pi r_o^{\,4}}{2}\left(1 - \frac{r_i^{\,4}}{r_o^{\,4}}\right) = \frac{\pi d_o^{\,4}}{32}\left(1 - \frac{d_i^{\,4}}{d_o^{\,4}}\right)$$



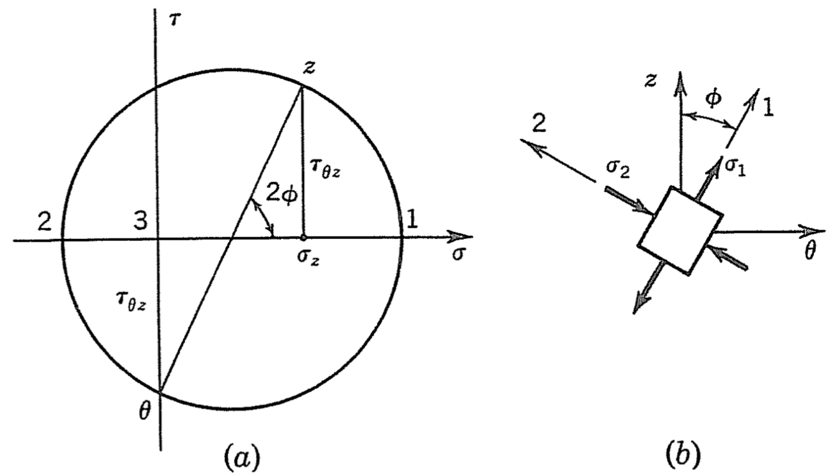
M_t
z
$\tau_{\theta z}$
θ
$|\sigma_1| = |\sigma_2| = |\tau_{\theta z}|$
A
σ_2
σ_1
Direction of maximum tension
Direction of maximum compression
B
45°
45°
M_t



τ
z
$\tau_{\theta z}$
2
1
O
σ
$\tau_{\theta z}$
θ



P
M_t
z
θ
M_t
(a)
P
+
=
Combined state of stress
(b)
M_t
P
$\tau_{\theta z} = \frac{M_t r_o}{I_z}$
$\sigma_z = \frac{P}{\pi r_o^2}$
σ_z
$\tau_{\theta z}$
+
=
(c)



τ
z
$\tau_{\theta z}$
2ϕ
2
3
1
σ
σ_z
$\tau_{\theta z}$
θ
(a)
z
ϕ
1
2
σ_2
σ_1
θ
(b)