Métodos de Energia

Prof. F.A.Rochinha – Julho de 2022

Ref. e ilustrações : An Introduction to the Mechanics of Solids . Crandall et. All. – 2.6, 5.8, 6.8, 7.8

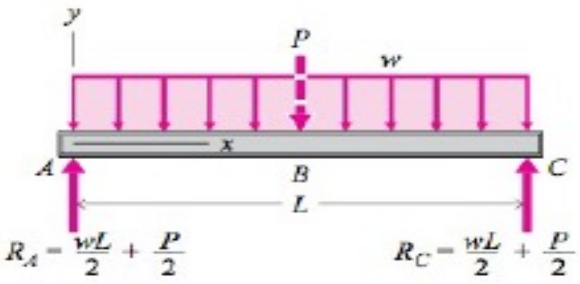


Figure 8-30

Example Problem 8-19 Determine the deflection at the center of a simply supported beam of constant cross section and span L carrying a uniformly distributed load w over its entire length.

SOLUTION

The deflection is required at a point where there is no unique point load. Thus, a dummy load P is introduced at the center of the beam in the direction of the desired deflection. In the free-body diagram of Fig. 8-30, the dashed force P represents the dummy load. The moment equation is

$$M_r = M_w + M_P = \frac{wLx}{2} - \frac{wx^2}{2} + \frac{Px}{2} - P\left\langle x - \frac{L}{2}\right\rangle^1$$

where the quantity $\langle x - L/2\rangle^1$ is zero for all $x \leq L/2$ (see Section 8-5). The partial derivative of M_r with respect to P is

$$\frac{\partial M_r}{\partial P} = \frac{x}{2} - \left\langle x - \frac{L}{2}\right\rangle^1$$

The dummy force P is equated to zero after the partial derivative is taken, and the deflection is given by

$$\begin{aligned} EIv &= \int_0^L M_r \frac{\partial M_r}{\partial P} dx \\ &= \int_0^L \left(\frac{wLx}{2} - \frac{wx^2}{2}\right)\left(\frac{x}{2} - \left\langle x - \frac{L}{2}\right\rangle^1\right) dx \end{aligned}$$

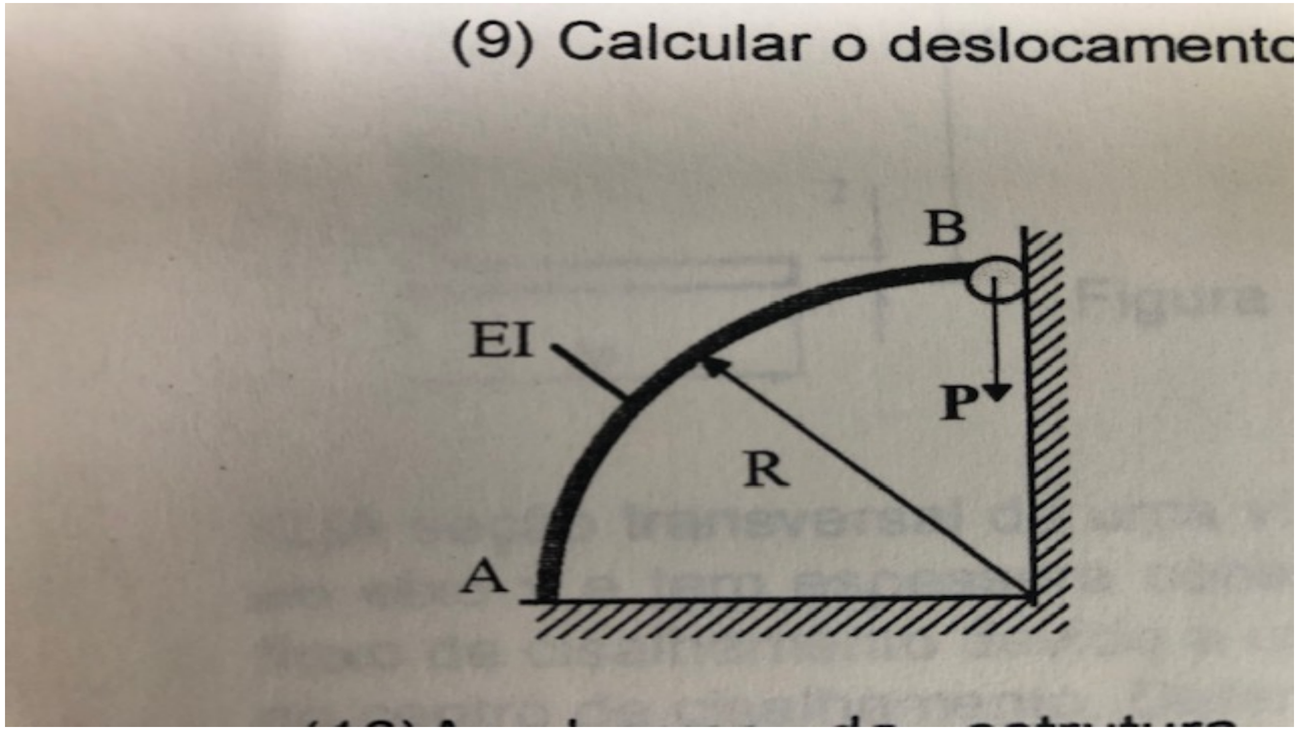
which, for ease of integration, can be written as

$$EIv = \frac{w}{4}\int_0^L (Lx^2 - x^3)dx + \frac{w}{4}\int_{L/2}^L (L^2x - 3Lx^2 + 2x^3)dx = \frac{5wL^4}{384}$$

Since the deflection is positive, it is in the direction of the force,

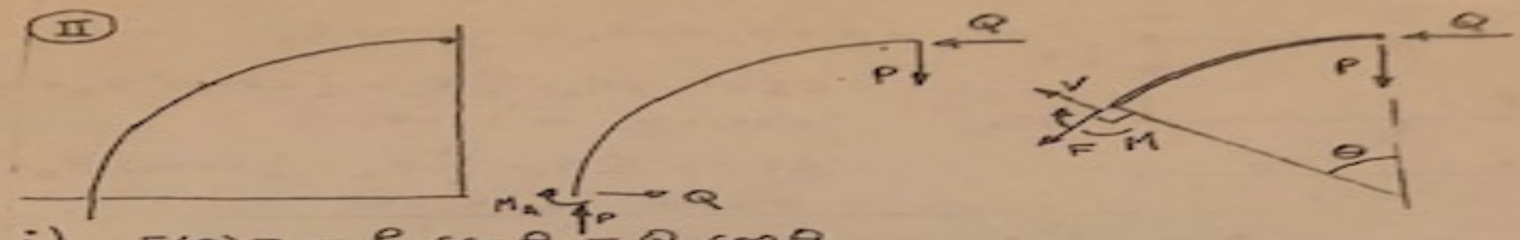
$$v = \frac{5wL^4}{384EI} \downarrow \qquad \textbf{Ans.}$$

This result corresponds to case 7 of Table B-19 (Appendix B).


(9) Calcular o deslocamento

Problema 4 da lista

II



i) $F(\theta) = -P \,\text{sen}\,\theta - Q\cos\theta$

$V(\theta) = P\cos\theta - Q\,\text{sen}\,\theta$

$M(\theta) = -PR\,\text{sen}\,\theta + QR(1-\cos\theta)$

ii) $$U = \frac{1}{2}\int_0^{\pi/2} \frac{R^3}{EI}\left[-P\,\text{sen}\,\theta + Q(1-\cos\theta)\right]^2 d\theta \; +$$

$$+ \frac{1}{2}\int_0^{\pi/2}\frac{R}{EA}\left(P\,\text{sen}\,\theta + Q\cos\theta\right)^2 d\theta$$

$\int_0^{\pi/2}\text{sen}^2\theta\, d\theta = \int_0^{\pi/2}\cos^2\theta\, d\theta = \frac{\pi}{4}$; $\int_0^{\pi/2}\text{sen}\,\theta\cos\theta\, d\theta = \frac{1}{2}$; e

$\int_0^{\pi/2}\text{sen}\,\theta\, d\theta = \int_0^{\pi/2}\cos\theta\, d\theta = 1$ $\therefore$

$$U = \frac{1}{2}\frac{R^3}{EI}\left[\frac{\pi}{2}Q^2 + \frac{\pi}{4}(P^2+Q^2) - 2PQ - 2Q^2 + \frac{1}{2}2PQ\right] +$$

$$+ \frac{1}{2}\frac{R}{EA}\left[\frac{\pi}{4}(P^2+Q^2) + \frac{1}{2}2PQ\right]$$

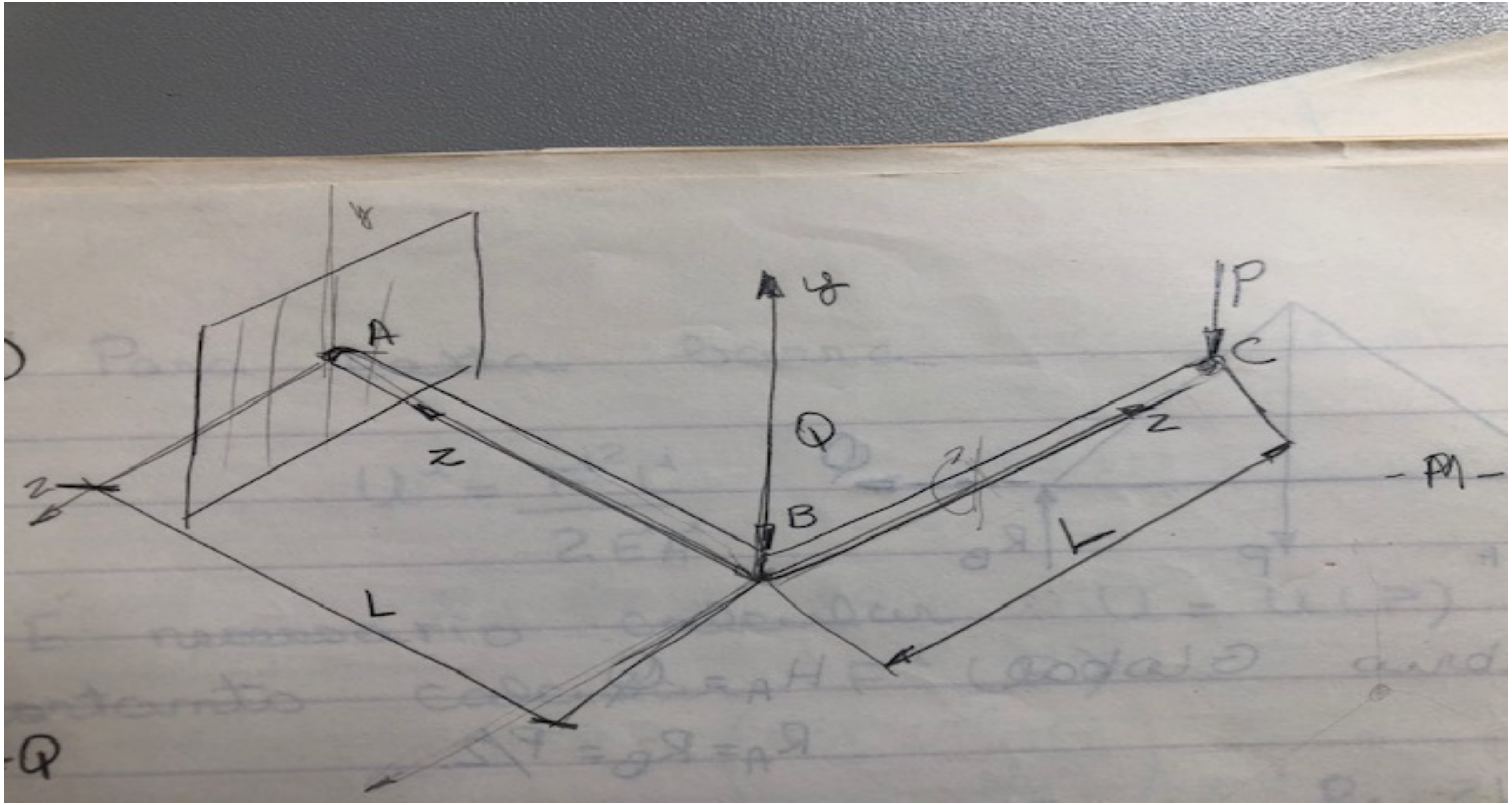
$$\boxed{U = \underbrace{\frac{R^3}{2EI}\left[\frac{\pi}{4}(P^2+3Q^2) - PQ - 2Q^2\right]}_{\text{flexão } (U_M)} + \underbrace{\frac{R}{2EA}\left[\frac{\pi}{4}(P^2+Q^2) + PQ\right]}_{\text{compressao } (U_F)}}$$

iii) $\delta_H = 0 \Rightarrow \frac{\partial U_M}{\partial Q} = 0 \rightarrow \frac{3\pi Q}{2} - P - 4Q = 0 \Rightarrow$

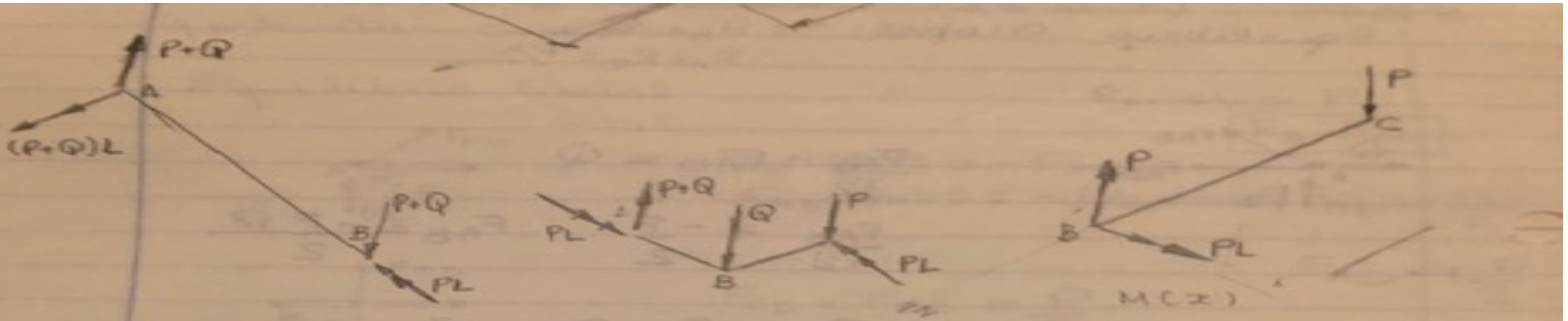
$$\boxed{Q = \frac{2P}{3\pi - 8} = 1.404\,P}$$

$$\delta_V = \frac{\partial U_M}{\partial P} = \frac{R^3}{2EI}\left(\frac{\pi P}{2} - Q\right)$$

$$\boxed{\delta_V = \frac{R^3 P}{EI}\left(\frac{\pi}{4} - \frac{1}{3\pi - 8}\right) = 0.0835\frac{R^3P}{EI}}$$



Exercício 3 da lsita



Calcular o deslocamento em C e B e

$U = U_{AB} + U_{BC}$

$$U_{AB} = \underbrace{\int_0^L \frac{(P+Q)^2 x^2}{2EI_z}\,dx}_{\text{flexão}} + \underbrace{\int_0^L \frac{P^2 L^2}{2GI_x}\,dx}_{\text{torção}} + \underbrace{\ldots\ldots}_{}$$

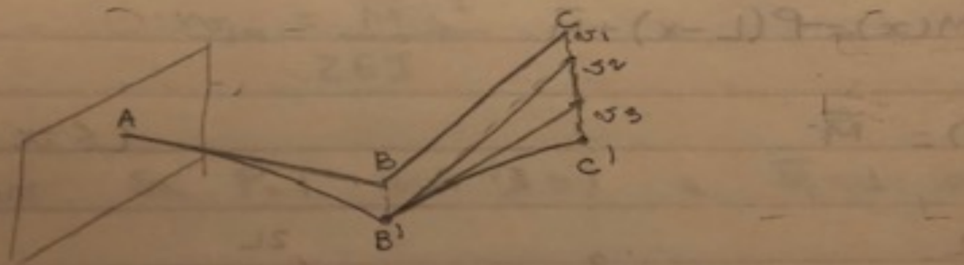
termos de de cortante desprezíveis em relação aos de

$$U_{BC} = \int_0^L \frac{P^2 (L-z)^2}{2EI}\,dz$$

$$v_B = \frac{\partial U}{\partial Q} = \int_0^L \frac{(P+Q)\,x^2 dx}{EI_z} = \frac{(P+Q)L^3}{3EI_z}$$

$$v_C = \frac{\partial U}{\partial P} = \int_0^L \frac{(P+Q)}{EI_z}x^2 + \int_0^L \frac{PL^2}{GI_x}dx + \int_0^L \frac{P(L-z)^2}{EI_z}$$

$$v_C = \underbrace{\frac{(P+Q)L^3}{3EI_z}}_{\text{flexão de AB } (v_1)} + \underbrace{\frac{PL^3}{GI_x}}_{\text{torção de AB } (v_2)} + \underbrace{\frac{PL^3}{3EI_z}}_{\text{flexão de BC } (v_3)}$$



Para caso: